STABILITY AND EQUILIBRIUM OF FLOATING BODIES

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BY

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PREFACE

The present treatise is an attempt to set forth briefly the principles underlying the stability and equilibrium of bodies floating partially or wholly submerged in water, and in air.

Hitherto published matter bearing on stability has for the most part been confined to ship forms. Submarines and aerial machines claim a more recent development, and problems relating to the stability and equilibrium of these bodies may be said to be still under investigation. It is necessary now to approach the subject in a more liberal manner, treating the bodies as subject to active as well as passive forces, and to call into requisition the principles of fluid pressure—whether liquid or gaseous—in their action upon bodies at rest and in motion. A knowledge of the salient features of rigid dynamics and hydromechanics is required in order to enable the reader to take a comprehensive view of the subject under discussion.

In the introductory chapter the author has endeavoured to set out the essential points bearing upon this phase of the subject, and in Chapter I. discusses generally definitions, the nature and conditions of equilibrium, and the important formulæ, in order to enable the reader to pursue without interruption the chapters dealing with specific types of body, which follow.

Chapter II. treats of the stability of ships, and an endeavour has been made to render the matter comprehensive without touching upon the historical side of the subject; those readers desirous of pursuing to the end the history and development of stability as applied to ships are referred to Sir E. J. Reed's classic work.

The consideration of floating docks is included in Chapter IV., and, so far as the author is aware, has not been dealt with hitherto in any published work.

Chapters III. and V., treating of submarines and air craft, indicate the manner in which the problem may be attacked without entering into what, to a large extent, could be only approximate data with reference to the forces acting on these bodies. It is hoped that the treatment of the subject may be found instructive to those interested in the study of submarine navigation and aerial flight.

Chapter VI. deals concisely with caissons.

The subject-matter for the most part has been derived from the author's notes culled over a period of intimate association with the scientific side of shipbuilding; the data and experimental results where given are reliable, and no effort has been spared to make the book trustworthy. Where necessary reference has been made to works and papers dealing with the subject.

The author is under obligation to Mr. Lyonel Clark, M.I.C.E., and to Mr. P. Hillhouse, B.Sc., for their patience and kindness in reading over the MS., and to Mr. A. E. Berriman, late Technical Editor of *Flight*, for reading the MS. dealing with air craft.

BERNARD C. LAWS.

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STABILITY AND EQUILIBRIUM OF FLOATING BODIES

INTRODUCTION

DEALING WITH THOSE PRINCIPLES OF THE ACTION OF FLUIDS WHICH DIRECTLY AFFECT THE EQUILIBRIUM AND STABILITY OF BODIES FLOATING AT THE SURFACE OF, OR ENTIRELY WITHIN, THE FLUID ITSELF.

As a matter of ordinary experience we know that fluids exert, and are capable of resisting, pressure.

Without going analytically into the question of classification, we may state broadly that fluids are of two kinds, viz. :—

Compressible and Incompressible,

preferring that the reader who may be desirous of obtaining a complete knowledge of the general properties of fluids should consult those published works which are devoted to the study of hydromechanics.

What affects us more in the present case is the consideration of the properties of water and air so far as concerns their ability to support and exert pressure on bodies, whether at rest or in motion, floating at the surface of the former or entirely within either. Consequently in this work the term "floating body" will generally embrace not only those bodies usually met with in practice, the normal position of which is at the surface of the water, viz., bodies of ship-shape form, floating docks, and caissons, but also those of more recent development specially designed for service in the air or beneath the water surface ; these latter embody respectively aeroplanes and airships, and submarines.

In order to understand the application of the principles governing the stability of these several types of body, it will be well to first establish, or record without proof, certain fundamental points, a knowledge of which will be found to be essential to the investigation.

Air, of course, is compressible; water, although not really incompressible, is generally regarded as such, because so great a force is required to compress it into a volume even a little less than that occupied under normal conditions.

Def.—A fluid is a substance which yields continually to the slightest stress in its interior, *i.e.*, it may be divided easily along any plane, provided the fluid be at rest.

If the fluid have motion or be of a viscous nature it still may be so divided if sufficient time be allowed. Hence when a fluid is at rest the tangential stress in any plane within it vanishes, and the resultant stress at any point in that plane will be entirely normal to it.

We may therefore conclude at once that in any fluid :—

(*a*) The pressure on a surface at any point is normal to that surface.

(*b*) The pressure on any elemental area is independent of the inclination of the plane containing that area, and is therefore the same in any direction.

The pressure at a point situated at a distance z below the surface of a fluid is given by

$$p = p_0 + w \, . \, z,$$

where :—p_0 = pressure at the fluid surface due to the atmosphere or otherwise

w = weight per unit volume of the fluid.

From this it follows that the pressure at all points in the same horizontal plane is the same.

The pressure at any point of a surface being perpendicular to the surface, it follows that the resultant pressure on a plane surface is equal to the sum of all the normal pressures, acts in a direction at right angles to the surface, and is given by :—

$$\iint (p_0 + w \, . \, z) \, da = \text{A} \, (p_0 + w \, . \, \bar{z}),$$

where :—A = area of the surface

$\bar{z}$ = distance of the centre of area below the fluid surface.

This resultant may be resolved into a vertical and horizontal component the directions of which lie in the same plane with the resultant.

If through any point in the boundary of the given surface a vertical line be drawn to the surface of the fluid, and this line be caused to traverse, parallel to itself, the whole boundary so

as to trace out the surface of a vertical cylinder enclosing a mass of fluid, then the reaction of the plane surface resolved vertically is equal to the weight of fluid in the cylinder and acts in a vertical line through the centre of gravity; the point at which this line cuts the plane is the "centre of pressure."

Consider a body of any form of surface floating freely at rest, and wholly or partially submerged. The resultant pressure of the fluid on the body may be resolved into a vertical pressure, and two horizontal pressures in directions at right angles to each other. Obviously the latter must be each zero, since there is no horizontal movement of the body, and the resultant vertical pressure must equal the weight of the body.

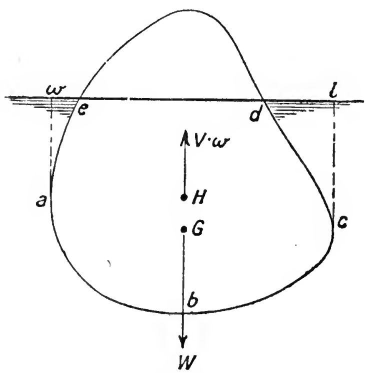


FIG. 1.

Suppose a vertical line touching the surface of the body to trace out a cylinder enclosing the immersed portion. The surface will be divided into two parts, on one of which—*abc*, Fig. 1—the resultant vertical pressure acts upwards, and on the other—*aedc*—downwards. The difference between these pressures is the resultant vertical pressure on the body, and is evidently equal to the difference between the weight of the fluid included in the cylinders *wabcl* and *waedlc*, *i.e.*, is equal to the weight of fluid displaced by the body, and must act upwards through the centre of gravity of the fluid displaced.

Therefore :—The weight W of the body must equal the weight V . w of the fluid displaced; and the centre of gravity G of the body and of the fluid displaced H must be situated in the same vertical line, where V is the volume of the fluid displaced.

These are the necessary and sufficient conditions of equilibrium of a body floating freely and at rest.

Def.—The centre of gravity of the displaced fluid is designated the centre of buoyancy of the body.

With constraint the conditions of equilibrium are altered. Suppose a body—*abcd*, Fig. 2—floating at the water line *bd* to be supported at *a* by a force or reaction R.

Let W and P denote the weight of the body and of the fluid displaced respectively. The resultant horizontal pressure of the fluid on the body is *nil,* and therefore the direction of R must be vertical, and we have :—

$$R = W - P$$

Hence, in the case of a body floating in equilibrium under constraint, the constraining force is vertical and is equal to the difference between the weight of the body and that of the fluid displaced.

The foregoing refers to the equilibrium of a body floating at rest. When we come to consider the conditions of equilibrium of a submarine—a body the function of which is to move at and below the surface of the water—we shall require to approach

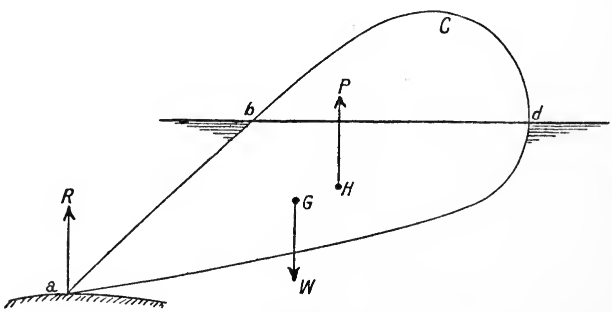


FIG. 2.

the questions of equilibrium and stability under new conditions, for upon the relative speed of the body and fluid and the angle at which the planes or fins superposed on the body and used for the purpose of stabilisation are presented to the direction of motion will depend the resistance experienced by the body, and the pressure on the planes ; as a result of which the body is enabled to assume certain required and definite positions in the fluid. It will therefore be necessary to consider concisely certain leading principles relating to the pressure of water on planes moving through the fluid or against which the fluid flows.

Similarly it will be necessary to know what is the nature of the pressure of a gaseous fluid on a body moving through it in order to be able to apply our knowledge to aeroplanes and air-

ships when we come to consider the equilibrium and stability of these bodies.

Suppose a thin lamina to be moving through water, the direction of motion being parallel to the moving surface. The resistance experienced will be purely frictional.

If the direction of motion be perpendicular to the moving surface, the resistance will be entirely head or direct resistance.

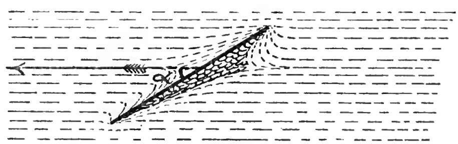


FIG. 3.

Suppose now the lamina to be moving in such a manner that the direction of motion is inclined to the moving surface, *i.e.*, that the surface is moving obliquely, the resistance will partake of both frictional and head.

In the case of motion which is either oblique or perpendicular to the moving surface, an eddying wake will form behind the lamina somewhat as indicated in Fig. 3 and 4 respectively, the arrows indicating the direction of motion of the body relative to the fluid. The dotted lines indicate the stream lines or paths pursued by the water, and the broken water produced immediately behind the lamina constitutes an element of resistance additional to that measured by the speed at which the lamina is moving.

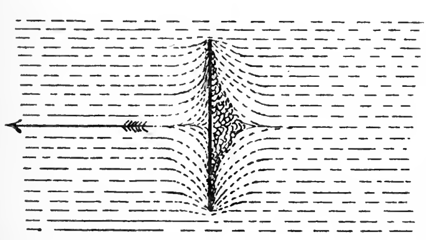


FIG. 4.

Beaufoy, in his experimental work in this department, ascertained that so long as the lamina was actually immersed, so that no surface disturbance of the fluid resulted, the resistance to motion was independent of the depth below the fluid surface, *i.e.*, was independent of hydrostatic pressure; and that the resistance per square foot in the case of a surface moving in a direction perpendicular to itself is 112 lbs., corresponding to a uniform speed of 10 ft. per second; for any other speed the resistance varies as the square of the speed.

Lord Raleigh, as the result of his experiments on surfaces relatively narrow, determined a formula connecting the oblique with direct resistance as follows :—

$$P_1 = \frac{P\,(4 + \pi)\,\sin a}{4 + \pi \sin a}.$$

where :—P_1 = normal pressure on the face of the plane.

P = pressure of a head due to the relative speed of the water and plane.

a = inclination of the direction of motion to the plane (see Fig. 3).

If $a = 90°$ then $P_1 = P$.

Beaufoy stated that, from his experiments, when $a = 90°$ the resistance was greater than P by some 16 per cent., from which it would appear that Lord Raleigh's formula takes no account of the negative pressure on the back surface of the lamina.

The position of the point of application of the pressure on a plane moving through water has been worked out by Lord Raleigh, and has been determined experimentally by Mr. Froude and others.

For a rectangular plate of breadth b and length l, of which l is perpendicular to the direction of motion, the distance x of the centre of pressure from the forward edge is expressed in the following formula given by Lord Raleigh :—

$$x = \frac{b}{2}\left\{1 - \frac{3 \cos a}{2\,(4 + \pi \sin a)}\right\},$$

where a is the angle made by the plane with the direction of motion.

There is obviously great difficulty in determining by experiment the value of x for small angles of inclination, but for angles greater than 10° the results may be considered as more trustworthy.

Froude's results for plates of certain proportionate dimensions are shown graphically in Fig. 5, and are taken from his Reports and Memoranda.

With reference to frictional resistance, the exhaustive experiments conducted by Mr. Froude and recorded in the

Reports of his researches are probably the most reliable. The following leading factors should be observed :—

(1) Frictional resistance is independent of the pressure between the fluid and solid.

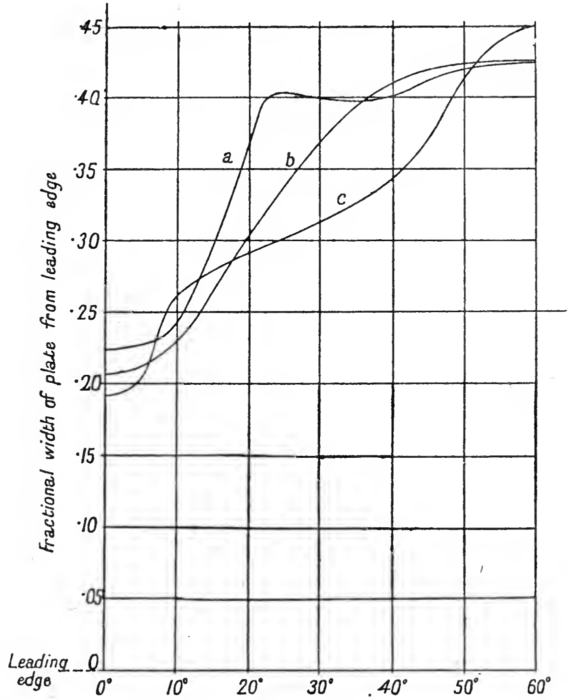


FIG. 5.—Diagram illustrating Froude's Experiments on Planes moving in Water.

Centre of Pressure Curves.

(*a*) $\frac{l}{b} = 2$—plane inclined about long edge.

(*b*) $l = b$—plane square.

(*c*) $\frac{b}{l} = 2$—plane inclined about short edge.

(2) Frictional resistance is proportional to the area of the surface presented to the fluid.

(3) Frictional resistance $\propto v^2$ (approximately)—where v is the velocity of the surface relative to the fluid.

(4) Frictional resistance depends upon the density of the fluid.

(5) Frictional resistance is sensibly affected by the length of surface in the direction of motion.

We may take $R = f \,.\, A \,.\, v^2$

where :—R = resistance

A = area of surface

f is a coefficient dependent upon the nature and degree of roughness of the surface.

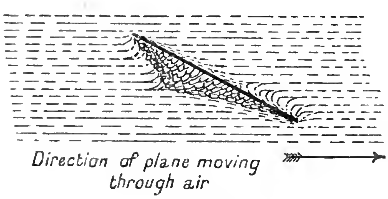


FIG. 6.

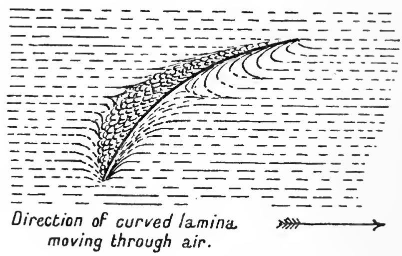


FIG. 7.

When a plane is moving through air the nature of the stream lines and the eddies behind the plane are of a very similar character to the phenomenon in the case of a lamina moving through water. Figs. 6 and 7 illustrate examples of a plane and curved lamina respectively.

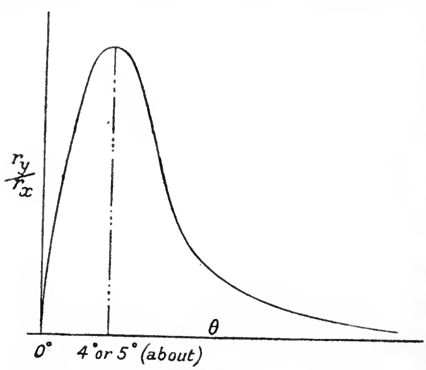


FIG. 8.

Our knowledge relating to air pressure and the position of the centre of pressure on surfaces moving through the air is not definite, although experiments have been and are still being carried out.

Mr. Mallock states * that he has made several series of experiments in air with reference to the forces acting on planes and curved laminæ. Taking r_x and r_y to represent the resistances parallel and perpendicular to the direction of

* Report of Advisory Committee on Aeronautics.

motion or flow, and θ to denote the angle of incidence,* his results are represented approximately by the diagram in Fig. 8, where the abscissæ denote angles of incidence and the ordinates the value of the ratio r_y/r_x.†

The maximum value of the ordinate of the curve appears to correspond with about four or five degrees of incidence, and may be taken at an average of 7—the values obtained during the experiments varying between 5 and 10.

With curved surfaces the angle of incidence—θ—has been reckoned relative to the chord of the arc, thus :—

FIG. 8A.

Mr. Mallock evidently considered his results as being very approximate.

Experiments carried out by Finzi and Soldate ‡ gave the following results :—

	Angle of incidence	0°,	5°,	10°,	15°.
Ratio r_y/r_x for	Flat surface	—	11·3	5·6	3·7
	Curved surface§	1·4	10·5	8·9	5·2

The most reliable work with reference to the surface friction of thin plates in air, according to Dr. Stanton,‡ appears to have been done by Zahm (see *Philosophical Magazine*, Vol. 8, 1904), who experimented on boards of various lengths placed in an air channel with an air current varying from five to twenty-five miles per hour. The results generally show that the resistance per square foot of area $\propto v^{1\cdot85}$.

Compared with Froude's experiments on plates in water (British Association Report, 1872), this leads to the conclusion that under similar conditions the resistance in the two media due to surface friction $\propto$ densities.

* In this work the angle of incidence is taken to be the angle at which the air meets the plane.

† Ratio of *lifting* to *propulsive* force in flying machines.

‡ *Engineering*, March 17th, 1905.

§ θ must be negative if r_y is to be zero with the curved surface.

If v = velocity of the wind in feet per second
and P = direct pressure in lbs. per square foot,
then $P = \cdot 0023\ v^2$ approximately, and the normal pressure of the plane for an angle of incidence θ, $= P \sin \theta$.

Generally the formula for P may be taken as $P = csv^2$, and the normal pressure for small angles of incidence $R = c_1 sv^2\theta$, where c and c_1, are coefficients dependent upon the "aspect ratio"* of the surface, and s is the area of the surface.

The mean value of c may be taken as $\cdot 08$, and c_1 as about four times this value for angles up to $7\frac{1}{2}$ degrees, and aspect

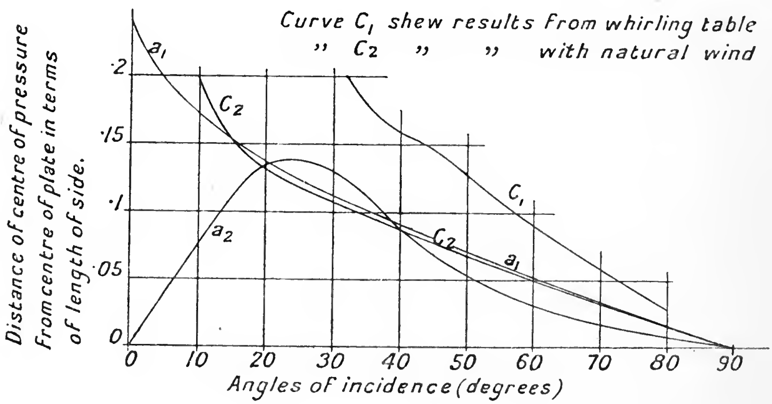


FIG. 9.

ratio 5 to 6 (M. Eiffel's experiments), where s is in square metres, v in metres per second, and R is in kilogrammes.

The point of application of the resultant pressure, *i.e.*, the centre of pressure, on a plane inclined to the direction of the air current, has been investigated by Dines (Proceedings Royal Society, 1891), and his results are shown in the above diagram (Fig. 9, curves c_1 and c_2).

M. Eiffel has carried out experiments on plane and curved surfaces, and his results are shown approximately in the same figure by the curves a_1 and a_2, the angles of incidence relating to the curved surface being considered with respect to the chord of the arc, and the curve a_2 indicates the projection of the actual centre of pressure on the chord. It will be seen that the centre

* Ratio of "span" or transverse dimension to "Fore and Aft" dimension.

of pressure for zero angle of incidence is at the centre of the arc, recedes from the centre for small angles, and after reaching a maximum distance approaches the centre again as the angle of incidence is increased, coinciding with the centre of arc at 90°.

The deviation of the position of the centre of pressure from the centre of the plate or lamina may be explained as follows :—Suppose a plane AB moving obliquely against the air in the direction indicated by the arrow (Fig. 9A), so that A is the forward moving edge. The air is compressed in the vicinity of A, while at the back edge B negative pressure results—this is indicated by the pressure curve cc_1, the ordinates of the curve being drawn perpendicular to AB. Behind the plane there is negative pressure, indicated by the curve dd_1, and the result of the combined action of the negative and positive pressures is that the centre of pressure for angles of incidence less than 90° is situated nearer the forward edge A than the back edge B.

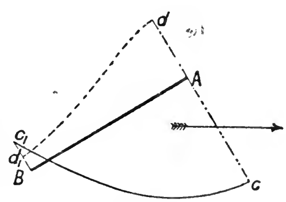


FIG. 9A.

CHAPTER I

GENERAL CONSIDERATIONS

THE following proposition is important, inasmuch as it will be found to have a direct bearing on the equilibrium of floating bodies when we come to consider the question of their inclination to the normal position.

"If a body be cut by a plane which is made to turn through a very small angle about a straight line in the plane in such a manner that the volume cut off from the body remains the same, the straight line will contain the centre of area of the plane section."

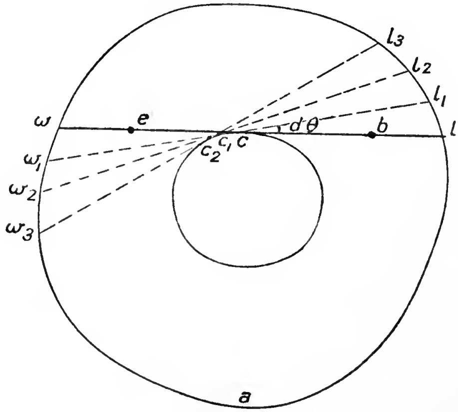


FIG. 10.

Let *wal*, Fig. 10, represent the body, and *wcl* the initial position of the cutting plane; let $w_1c_1l_1$ be the second position of the plane where the angle lcl_1 is very small and equal to $d\theta$. Consider any small area *da* containing the point *b* in the plane *wcl*, then since by hypothesis the volume cut off by the plane remains constant, therefore the wedges wcw_1 and lcl_1 are of equal volume, where *c* represents the line of intersection of the plane *wcl* and $w_1c_1l_1$.

The volume of the wedge lcl_1 is given by $d\theta\int cb \times da$.

Similarly the volume of the wedge wcw_1 is given by $d\theta\int ce \times da$, the integrations being taken over the areas *cl* and *cw* respectively.

Therefore $d\theta\int cb \times da - d\theta\int ce \times da = o$, which is the condition that *c* contains the centre of area of the plane *wcl*.

It will be seen that for a further small angular displacement the straight line representing the intersection of the planes wcl, $w_1c_1l_1$ will not necessarily contain the centre of area of the plane $w_1c_1l_1$, unless the body be symmetrical about the plane wcl, and a plane through c perpendicular to it; consequently the line of intersection of the planes $w_2c_2l_2$ and $w_1c_1l_1$ will be at c_1, the centre of area of the plane $w_1c_1l_1$, a position different from c, and so on for further small angular displacements.

The surface cc_1c_2 . . ., containing all such points, will be the envelope of all planes cutting off the constant volume from the body, and the point of contact of any plane with the envelope will be the centre of area of that plane.

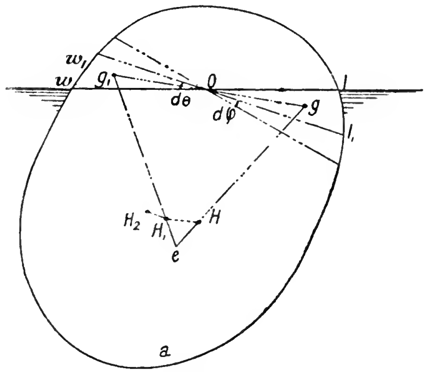


FIG. 11.

It is obvious that if the surface of the body be closed that of the envelope will also be closed.

If now we take the converse problem, in which, instead of assuming the plane to move and the body fixed, we consider the plane wcl as fixed and the body to experience angular displacement, then, in order for the volume cut off from the body by the plane to be constant, the surface cc_1c_2 . . . will always be in contact with the plane wcl as the body inclines. The case resembles that of a body floating partially submerged in a fluid and with a constant volume of displacement; the plane wcl is termed the "plane of flotation" and all other planes, such as $w_1c_1l_1$, are also so designated; and the envelope cc_1c_2 . . . is termed the "surface of flotation."

The motion of the body might therefore be determined by considering the surface cc_1c_2 . . . as rolling in contact with the plane wcl, and if for the moment we disregard the effect of the acting forces, such as the weight of the body and the resistance of the fluid to the motion of the body, then, since the axis of rotation at any moment passes through the point of contact of the particular plane of flotation and the surface of

flotation, the fixed plane *wcl* would be the locus of the axis of rotation in space and the surface of flotation the locus relative to the body.

In practice, however, this ideal condition does not exist on account of the effective and impressed forces acting on the body, with the result that a constant volume of displacement is not preserved, and consequently the surface of flotation undergoes modification, and pure rolling of the latter in contact with the plane of flotation is not nearly realised. We shall return to this point later in the present chapter.

Reverting to the case of a body floating partially submerged in a fluid the surface of which corresponds to the fixed plane *wcl* and the volume of fluid displaced—assumed constant—to the volume *wal*, let H, Fig. 11, be the centre of gravity of the fluid displaced, and let g, g_1 be the centres of gravity of the wedges lol_1, wow_1 respectively for the angular displacement $d\theta$. If e be taken in gH, such that

$$\frac{e\mathrm{H}}{g\mathrm{H}} = \frac{\text{volume } lol_1}{\text{volume } wol_1a},$$

then e is the centre of gravity of the volume wol_1a.

Join eg_1 and draw HH_1 parallel to gg_1 to cut eg_1 in H_1, then :—

$$\frac{e\mathrm{H}_1}{g_1\mathrm{H}_1} = \frac{e\mathrm{H}}{g\mathrm{H}} = \frac{\text{volume } lol_1}{\text{volume } wol_1a} = \frac{\text{volume } wow_1}{\text{volume } wol_1a},$$

which is the condition that H_1 is the centre of gravity of the new volume w_1al_1. Similarly, for a further rotation $d\varphi$ the position H_2 of the new centre of gravity of fluid displaced may be found.

The surface containing all such points as H, H_1, H_2 . . . for any and all angular displacements of the body is termed the "surface of buoyancy."

If the body be rotated in any particular planar direction successively through definite angles, and the corresponding centres of buoyancy H, H_1, H_2 . . ., as indicated above, be projected perpendicularly on to a plane parallel to the direction plane, the locus of all such projections is termed the "curve of buoyancy," corresponding to that specific direction.

Similarly the locus of the projection of the centre of flotation on a plane parallel to that of the direction of rotation of the body is termed the "curve of flotation."

Such curves are indicated by B and F in Fig. 12.

Now, since $d\theta$ is infinitely small, gg_1 may be taken as being coincident with aoc, and therefore HH_1 is parallel to aoc; but HH_1 will be infinitely small, since its value depends upon the volume of the wedges aoa_1, coc_1, and may therefore be considered as coinciding with the locus or path of the centre of buoyancy at H, *i.e.*, with the tangent plane to the surface of buoyancy at H, and this argument will be true whatever the angular position of the body relative to the normal may be; in other words :—

"The tangent plane at any point on the surface of buoyancy is parallel to the plane of flotation at which the vessel will float with that particular point as centre of buoyancy; and the

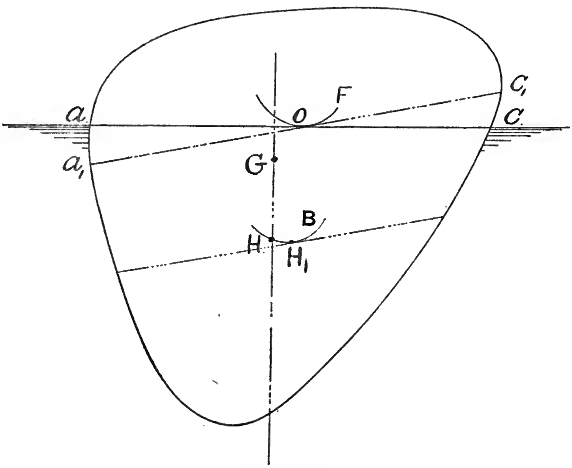


Fig. 12.

plane drawn parallel to the above-mentioned tangent plane and touching the surface of flotation will be the plane of flotation corresponding to this centre of buoyancy."

This is illustrated in Fig. 12, where a_1c_1 is the plane of flotation corresponding to the centre of buoyancy H_1, F and B respectively representing the surfaces of flotation and buoyancy.

The nature of the surface (or curve) of flotation F will depend upon the relative position of the centre of area o of the initial water plane ac, and the intersections of the various planes of flotation with ac according to the direction in which the body is inclined.

If the form of the body in the vicinity of the initial water plane be such that it is fuller above than below the water plane, then it is evident, since the wedges of immersion and emersion must be equal in volume, that for inclinations in the clockwise direction the new water planes, such as a_1c_1, Fig. 12, will intersect ac to the right of o, and for inclinations in the anti-clockwise direction the intersection would lie to the left of o; consequently the surface of flotation F would be convex downwards, as shown.

On the other hand, if the form of the body were fuller below the initial water plane ac than above it; the intersections of the new water planes with ac would lie to the left of o for clockwise inclinations of the body and to the right of o for anti-clockwise inclinations in order to satisfy the condition that the volumes of the wedges of immersion and emersion are equal. The surface of flotation will consequently be convex upwards, as indicated in Fig. 13.

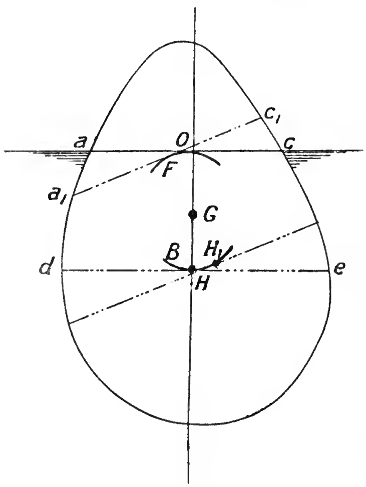


Fig. 13.

It is noteworthy that, whereas the downward or upward convexity of the surface or curve of flotation is dependent upon the form of the body, the surface of buoyancy B is invariably convex downwards—at least for the *first* angular displacements of the body from its normal position of equilibrium; and for the reason that a new volume of displacement to the water line a_1c_1 is obtained by transferring the wedge aoa_1 to coc_1 at a higher level, in consequence of which the new centre of buoyancy occupies a higher position relative, say, to the initial horizontal plane dHe drawn through H—the centre of buoyancy for the normal position of equilibrium of the body, Fig. 13.

If we were to trace the path of the centre of buoyancy through all the stages of inclination of the body from zero to 360°

it would be found that the locus is a closed surface—the nature of which will depend upon the form of the body—generally free from abruptness and possessing the property of having only two tangent planes which may be drawn parallel to any given plane.

The surface of flotation is also closed, and may possess, as it generally does, irregularities often in the nature of cusps due to non-uniformity or abrupt changes in the outward shape of the body.

The surface of buoyancy H and that of flotation F for a

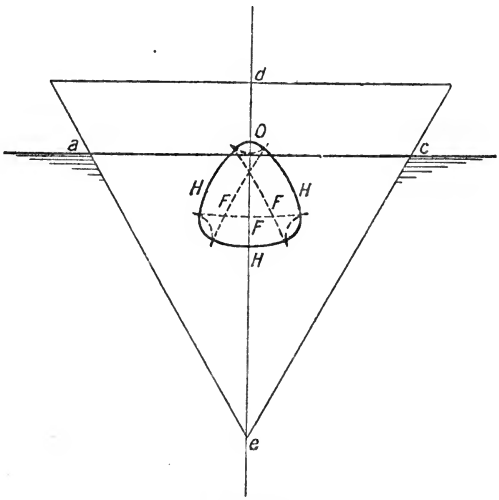


FIG. 14.

complete rotation of the body is illustrated in Fig. 14, which represents the case of a prismatic body the principal section of which is an equilateral triangle floating at a draught *oe*, equal to ·8*de*.

It was stated above that the nature or form of the path of the centre of buoyancy is dependent upon the form of the body; it may now be further stated that the character of the surface will be unaffected by an alteration or change of the form of that portion of the body which always remains immersed. This may easily be proved as follows :—

Consider the body *abcde* floating at the water plane *ac* to have its centre of buoyancy situated at H, Fig. 15, and let

the form of the immersed portion of the body be changed to abc_1de and be such that the part bc_1dc always remains below the surface of the water.

Let h denote the centre of volume of the portion $bcdc_1$, H, the centre of buoyancy, corresponding to the boundary $abcde$ and k the centre of volume of the portion abc_1de, then, if V and v respectively denote the volumes abc_1de and $bcdc_1$, we have :—

$$\frac{h\mathrm{H}}{hk} = \frac{\mathrm{V} - v}{\mathrm{V}}.$$

If we now take any other line radiating from h, such that

$$\frac{h\mathrm{H}_1}{hk_1} = \frac{\mathrm{V} - v}{\mathrm{V}},$$

then H_1 and k_1 represent the centres of volume or buoyancy respectively of $abcde$ and abc_1de for some one—the same—inclined position of the body, and it is obvious from the above relation of $h\mathrm{H}_1$ to hk_1 that the locus of k is similar to that of H, *i.e.*, the *character* of the surface of buoyancy is unaffected by changing the immersed portion of the body.

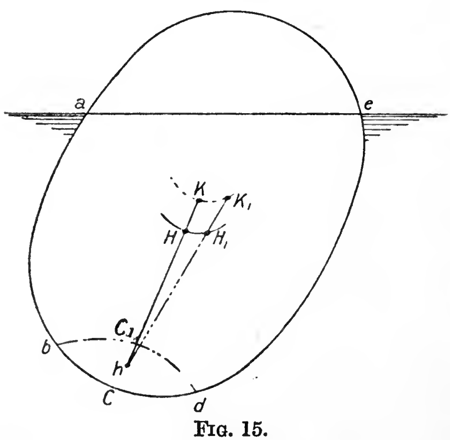


FIG. 15.

Referring to Fig. 12, let G denote the centre of gravity of the body and H_1 the centre of buoyancy referred to the water line or plane of flotation a_1oc_1, the inclination of the body being θ. If, through H_1, a normal be drawn to the surface of buoyancy B, this will be the line of action of the resultant pressure or buoyancy of the water, and if it pass through G the angle θ will represent a position either of stable or unstable equilibrium. Assuming the initial position of the body in which H is the centre of buoyancy to be one of stable equilibrium, HG may be termed the first normal to the surface of buoyancy. If now the body be inclined so that $\mathrm{H}_1\mathrm{G}$ is the next succeeding normal from G to the surface of buoyancy

it will be found that this position is one of unstable equilibrium, and by further inclining the body until another position is reached at which a normal may be drawn from G to the surface, this position will be one of stable equilibrium, and so on alternately. Indeed, it is a mathematical law that the positions of stable and unstable equilibrium occur alternately.

All such positions may be determined if, having given the surface of buoyancy, normals be drawn to that surface from the centre of gravity of the body; such normals, when brought into the vertical position by inclining the body, will give the actual positions of equilibrium, the angle at which

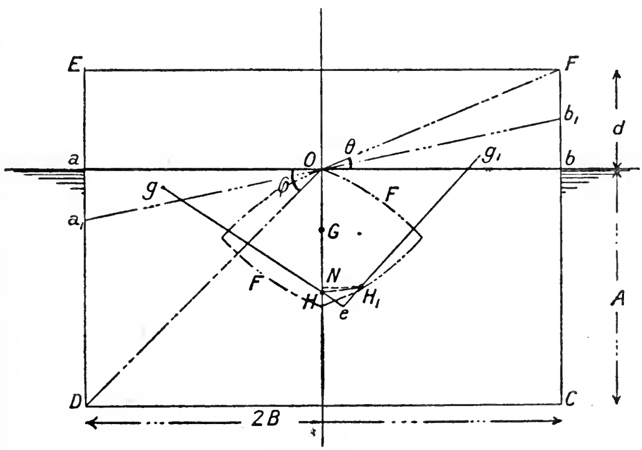


FIG. 16.

the normal is inclined to the initial vertical plane or line through the centre of gravity and centre of buoyancy being the angular displacement of the body corresponding to the position of equilibrium represented by that particular normal.

When we come to consider the stability of floating bodies of ship-shape form it will be found that there are generally at least two positions of equilibrium which the body may assume in passing from zero to 180 deg. of inclination; the one—the initial or upright—position being that of stable equilibrium, the other—at which the stability vanishes—being a position of unstable equilibrium. There may be a third position, as in the case of vessels sometimes met with in practice which, when light or badly loaded, may be initially unstable; such

vessels will loll over to one side or the other until a position of angular displacement, usually small, is reached at which stability is acquired. If the vessel be further inclined an angle will ultimately be reached at which she will be again unstable.

We will now consider the simple case of a homogeneous prismatic body of rectangular shape, Fig. 16, floating at the water line ab, with its centre of gravity G and centre of buoyancy H situated in the same vertical line with o, the centroid of the water plane when floating in the initial (upright) position.

It will be evident that for all inclinations of the prism up to that at which the point F or E—or D or C if φ be less than θ—is brought to the surface of the water, all the water planes will pass through o, and therefore the envelope of the water planes, *i.e.*, the "surface of flotation," reduces to the point o for all inclinations up to θ; for further angular displacements the nature of the envelope changes from a point to a curved surface until a position is reached at which the envelope is again represented by a point, and so on alternately throughout a complete rotation of the body. The curve (or surface) of flotation is indicated by the broken curve FF . . . in the figure.

That the surface of buoyancy is paraboloidal and the curve of buoyancy a parabola may be proved as follows :—

Let the body suffer an angular displacement so that a_1b_1 becomes the new plane of flotation and H_1 the new centre of buoyancy corresponding to a_1b_1. Then if g and g_1 are the centres of gravity of the wedges aoa_1, bob_1 respectively we have, as explained on p. 14,

$$\frac{eH_1}{eg_1} = \frac{\text{volume } bob_1}{\text{volume } ab\text{CD}} = \frac{d \times \frac{B}{2}}{2\,B \times A} = \frac{d}{4\,A}.$$

Draw H_1N perpendicular to HO, then—

$$\frac{eH_1}{eg_1} = \frac{HH_1}{gg_1} = \frac{H_1N}{\frac{2}{3} \times 2\,B} = \frac{d}{4\,A},$$

$$i.e.,\ H_1N = \frac{d \times B}{3\,A}.$$

Now $HN = H_1N \tan NH_1H = H_1N \tan g_1ob =$

$$\frac{d \times B}{3\,A} \times \frac{d}{2\,B} = \frac{d^2}{6\,A},$$

$$\therefore \quad H_1N^2 : HN = \frac{d^2B^2}{9\,A^2} : \frac{d^2}{6\,A} = \frac{2\,B^2}{3\,A} : 1.$$

Hence, if H_1 be considered with relation to rectangular axes through H, of which Ho is the axis of x, and a straight line perpendicular to it, the axis of y, we have

$$y^2 = \frac{2\,B^2}{3\,A} \cdot x,$$

or the curve of buoyancy is a parabola, and the surface of which this curve is a principal section is paraboloidal.

The case of a vessel of circular cylindrical section is interesting. Let the cylinder, the centre of which is C, Fig. 17, float initially at the water line ab, and let H represent the centre of buoyancy referred to the normal position of equilibrium; with C as centre sweep two circles of radii Co and CH.

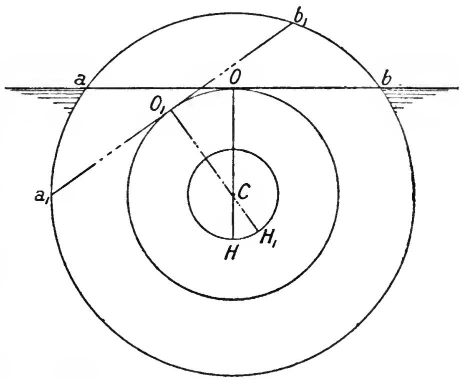


Fig. 17.

It will be seen that, as the vessel inclines, all water lines, as, for example, a_1b_1, will be tangential to the circle of radius Co. This circle, therefore, is the curve of flotation.

Again, since the centre of buoyancy for the constant displacement will be situated at a constant distance equal to OH below the water line, the circle of radius CH is the curve of buoyancy.

It will be noticed that as the draught at which the vessel may float increases from zero to the radius and thence to the diameter of the cylinder, the curve of flotation diminishes from a circle of radius equal to that of the cylinder to a point at the centre of the latter, and thence increases to a circle of the same radius as the cylinder. The curve of buoyancy, on

the other hand, diminishes from a circle of radius equal to that of the cylinder to a point at the centre C as the draught increases from zero to the diameter of the cylinder.

That the curves of flotation and buoyancy in the case of a prismatic vessel of triangular section are hyperbolas may be proved as follows :—

Let *abc*, Fig. 18, represent the prism floating at the water

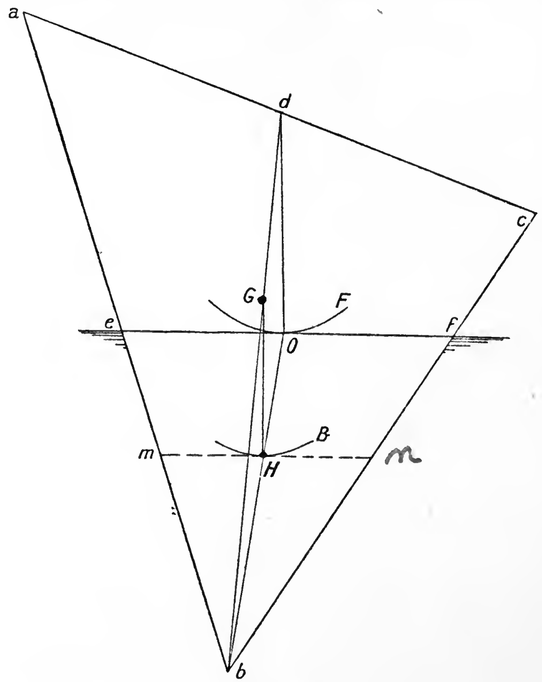


Fig. 18.

line *ef* with its edges horizontal; take *d* and *o*, the middle points of *ac* and *ef* respectively. Join *bd* and *bo* and make

$$bg = \frac{2}{3}\,bd, \text{ also } bH = \frac{2}{3}\,bo.$$

H and G are the centres of gravity respectively of the triangles *bef* and *abc*, and therefore represent the centres of buoyancy and gravity of the vessel.

Now the area *bef* is assumed to be constant, *i.e.*, *ef* and the successive water lines e_1f_1, etc., enclose with *ba*, *bc* a constant area; therefore *ef*, e_1f_1, etc., will be tangential at their middle

points o, o_1, etc., to a hyperbola of which the asymptotes are ba and bc. This hyperbola is the curve of flotation F.

Again, through H draw $m\mathrm{H}n$ parallel to ef; then :—

$$\frac{\text{area } bmn}{\text{area } bef} = \frac{bn^2}{bf^2} = \frac{b\mathrm{H}^2}{bo^2} = \frac{2^2}{3^2},$$

$$\text{or area } bmn = \frac{4}{9} \cdot \text{area } bef = \text{constant} = k;$$

hence all lines, such as $m\mathrm{H}n$ drawn through the centre of buoyancy parallel to the water line, cut off a constant area k, and will therefore be tangential at H, H_1, etc. (the middle points of mn, m_1n_1, etc.), to a hyperbola the asymptotes of which are ba and bc.

This hyperbola is the curve of buoyancy B.

If the figure represents a position of stable equilibrium of the body, then HG being vertical will be perpendicular to ef; but HG is parallel to od, therefore od will be perpendicular to ef—that is, the straight line joining the middle point of the side (a face) ac of the prism and that (the centre of gravity) of the water line must be perpendicular to the water line.

In the foregoing we have endeavoured to explain as concisely as possible compatible with lucidity the principal features pertaining to the equilibrium of floating bodies; we have shown how the curves or surfaces of flotation and buoyancy may be obtained and their relation to the angular displacement of the body as the latter becomes more or less inclined from the initial position of equilibrium.

It remains now for us to consider the conditions which hold in the case of a body inclined from the normal position of rest; whether in such a case the body is likely to return to its initial state or has a tendency to augment that inclination —in other words, we have to determine whether the initial position is one of stable or unstable equilibrium.

It is obvious that in taking up a new position the movement will be such as generally to partake of angular displacement together with motion in the vertical direction. With reference to the latter, it is evident in the case of a body floating partially submerged that there will be a resultant effective force equal to the difference between the weight of water displaced and the weight of the body, which will cause the body to rise and fall or oscillate in the vertical direction. Such a condition

is one of stable equilibrium, and the determination of the time of oscillation, being of interest, will be referred to later.

Such vertical motion, in the case of a body inclined in still water, may generally be disregarded, and we are left with the problem of angular displacement on the supposition that the weight of fluid displaced remains unchanged as the body inclines.

We will proceed to determine what are the conditions of equilibrium.

Consider a body, Fig. 19, floating at rest in still water to suffer a *small* angular displacement θ.

Let ac be the initial and a_1c_1 the ultimate water line corresponding to the new condition, and let H and H_1 represent the corresponding centres of buoyancy.

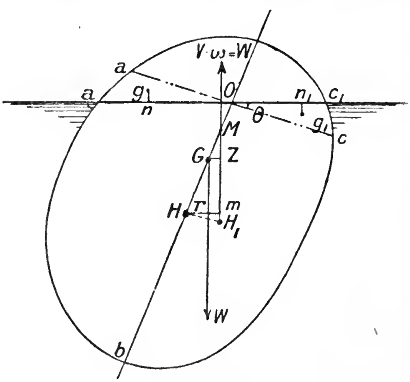


FIG. 19.

The new condition is tantamount to replacing the buoyancy of the wedge aoa_1 by that of the wedge coc_1, of which the centres of gravity are g and g_1 respectively; the volumes of the wedges are equal, since the weight of fluid displaced is constant.

Let v denote the volume of either wedge, V the volume of fluid displaced by the body, W the weight of the body, and w the weight of unit volume of the fluid.

We have shown, Fig. 11, p. 13, that

$$\frac{HH_1}{gg_1} = \frac{v}{V}.$$

Also we know that

$$\frac{HH_1}{gg_1} = \frac{Hm}{nn_1} = \frac{HM \sin\theta}{nn_1},$$

where Hm and nn_1 are the horizontal projections of HH_1 and gg_1 respectively, since HH_1 is parallel to gg_1.

Therefore $$\frac{HM \sin\theta}{nn_1} = \frac{v}{V},$$

or $$HM = \frac{v \,.\, nn_1}{V \cdot \theta}, \text{ since } \theta \text{ is small.}$$

Let the ordinates of the water plane a_1oc_1 be referred to the axis of x in the direction a_1oc_1 and to the axis of y at o—the centre of gravity of the water plane—viz., the intersection of the water planes aoc, a_1oc_1, which is, moreover, perpendicular to the plane abc.

Consider a vertical slice or lamina of the wedges aoa_1, coc_1 of thickness dy, and let x_1 represent any ordinate oa_1, we have :—

$$\text{Moment of lamina of wedge } aoa_1 \text{ about } o = \frac{x_1{}^2\theta}{2} \cdot dy \times \frac{2}{3} \cdot x_1$$

$$= \frac{x_1{}^3\theta}{3} dy.$$

$$\text{Moment of wedge } aoa_1 \text{ about } o = \int \frac{x_1{}^3\theta}{3} \cdot dy = \theta \int \frac{x_1{}^3}{3} \cdot dy = v \cdot on.$$

Similarly :—

$$\text{Moment of wedge } coc_1 \text{ about } o = \theta \int \frac{x_2{}^3}{3} \cdot dy = v \cdot on_1,$$

where x_2 represents any ordinate oc_1.

Therefore the sum of these moments $= v\ (on + on_1) = v \cdot nn_1$

$$= \theta \int \frac{x_1{}^3}{3} \cdot dy + \theta \int \frac{x_2{}^3}{3} \cdot dy$$

$$= \theta \int \frac{x^3}{3} \cdot dy \quad \ldots, \text{ suppressing the accents of } x,$$

the integration being taken over the whole water plane area,

$$\text{or } v \cdot nn_1 = \theta \cdot \text{I},$$

where I is the moment of inertia of the water plane area referred to the axis of y through o, its centre of area.

Hence $$\text{HM} = \frac{\theta \cdot \text{I}}{\text{V} \cdot \theta} = \frac{\text{I}}{\text{V}}.$$

The point M is termed the "metacentre," and may be defined as the point at which for *small* angles of inclination the line of action of the upward pressure of the water through the new centre of buoyancy intersects the initial vertical plane through H and G (the centre of gravity of the vessel).

The height of metacentre above the centre of buoyancy is therefore equal to the "moment of inertia of water plane area referred to the axis in that plane drawn through its centre of area perpendicular to the plane of angular displacement" divided by the "volume of fluid displaced."

By considering the metacentre geometrically as the "centre

of curvature of the surface of buoyancy at that point occupied by the centre of buoyancy H for the initial position of equilibrium," the expression $\frac{I}{V}$ may be defined as the radius of curvature of the surface of buoyancy at H.

For small angles of inclination θ it has been shown that the position of M is practically constant; this generally holds where the inclination does not exceed about 8 deg. from the initial position of rest. For any considerable angle of inclination φ the position of the point of intersection of the line of action of the buoyancy of the water with HG will vary, and may be designated M^1 to distinguish it from M the metacentre.

We will establish the conditions of equilibrium of the body in these two cases, viz.:—

(*a*) When θ is small and consequently M a fixed point.

(*b*) When φ is considerable and M^1 variable in position.

(*a*) Referring to Fig. 19, it is seen that when the body is inclined there is a mechanical couple $W \times Gz$ tending to restore it to its original position.

$$\text{Now } Gz = GM \sin\theta, \text{ and } GM = HM - HG = \frac{I}{V} - HG.$$

$$\text{Therefore restorative couple} = W\left(\frac{I}{V} - HG\right)\sin\theta$$

$$= W\left(\frac{I}{V} - HG\right)\theta, \text{ since } \theta \text{ is small.}$$

Now I, V and HG are calculable from the known shape and dimensions of the body, and the body will be in stable, neutral, or unstable equilibrium according as $\frac{I}{V}$ be greater than, equal to or less than HG.

(*b*) For a considerable angle of inclination φ the restorative couple will still be $W \times Gz$; draw Hrm parallel to Gz, *i.e.*, horizontally, then :—

$$Gz = Hm - Hr$$
$$= Hm - HG \sin\varphi.$$

Now $\frac{Hm}{nn_1} = \frac{v}{V}$, where v is the volume of either of the wedges aoa_1, coc_1.

Therefore $$Gz = \frac{v \cdot nn_1}{V} - HG \sin \varphi,$$

and the restorative couple is :—

$$W \times Gz = W \left(\frac{v \cdot nn_1}{V} - HG \sin \varphi\right).$$

This is "Attwood's fundamental formula of statical stability."

It is evident from the above that for the body initially to possess stable, neutral, or unstable equilibrium the position of the metacentre M must lie respectively above, coincide with, or fall below G, the centre of gravity of the body.

GM is designated the "metacentric height" or "initial stability" of the body, and is such that M is the limiting position to which the centre of gravity may rise without the body becoming unstable.

In the preceding investigation it has been assumed that when the body is inclined, the centre of buoyancy remains in the vertical plane *abc*, passing through HG; this is equivalent to assuming that the body is symmetrical about the plane *abc*, and will only be true when the section made by this plane with the surface of buoyancy is a principal section. When this is not the case, the above expression for restorative moment will still be true by considering that the projection of the line of action of the displaced fluid pressure on the vertical plane of displacement *abc* will intersect HG in a point M, which will be the centre of curvature of the normal section of the surface of buoyancy at H.

The foregoing consideration of the angular movement of a floating body is entirely statical, and has consisted of the determination as to whether the mechanical moment $W \times Gz$ would rotate or tend to rotate the body in the clockwise or anti-clockwise direction. This moment might be considered with relation to an axis through G—the centre of gravity of the body—perpendicular to the plane of angular displacement *abc*.

Considered dynamically, rotation will not take place about the horizontal axis through G unless the latter be a principal axis, on account of the forces introduced in consequence of the displacement of the body from its normal position. Moreover, unless HG passes through *o*, a rotation about G would cause a change in the quantity of fluid displaced, which would generally

have to be taken into consideration, and vertical oscillation would ensue.

If HG pass through o, then—at least for small angular displacements — if we consider rotation about G, o will move through an arc which is practically coincident with the plane of flotation, so that the quantity of fluid displaced remains unchanged, and the vertical movement of G may be disregarded. We can therefore neglect the work done in oscillating the body and consider only that done in producing angular displacement.

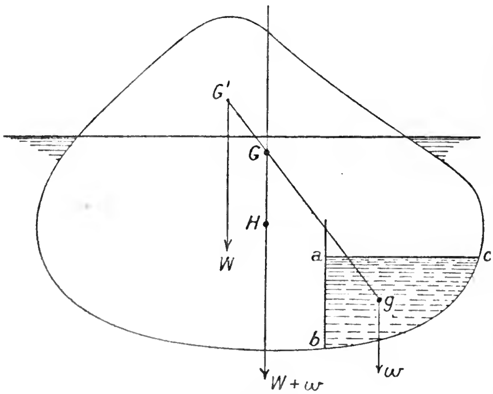


FIG. 20.

The mechanical couple tending to rotate the body at an angle θ is :—

$$W\left(\frac{v \,.\, nn_1}{V} - HG \sin\theta\right).$$

If the angle θ be increased or diminished by the amount $d\theta$, the work done during the change of inclination is :—

$$W\left(\frac{v \,.\, nn_1}{V} - HG \sin\theta\right) d\theta.$$

The whole amount of work done in inclining the body through an angle φ is :—

$$\int_o^\varphi W\left(\frac{v \,.\, nn_1}{V} - HG \sin\theta\right) d\theta.$$

in which $v \,.\, nn_1$ is a variable quantity.

If φ be small the expression for the amount of work done becomes :—

$$\int_o^\varphi W\left(\frac{I}{V} - HG\right)\theta \,.\, d\theta = W\left(\frac{I}{V} - HG\right)\frac{\varphi^2}{2}.$$

These formulæ will be referred to in the next chapter dealing with the stability of ship-shape bodies.

A case of very great importance, and one frequently met

with in practice, is that of a vessel containing a mobile fluid, *i.e.*, a fluid free to move as the vessel inclines from the normal position of equilibrium.

Let Fig. 20 represent a body of irregular shape floating freely and in equilibrium, and containing a quantity of fluid which is free to move.

Let W denote the weight of the vessel *only*, and G^1 its centre of gravity.

Let w denote the weight of the contained fluid, and g its centre of gravity.

and $W + w$ denote the weight of the vessel and fluid, and G their common centre of gravity.

Let H denote the centre of buoyancy.

Suppose the vessel to be inclined through a small angle θ,

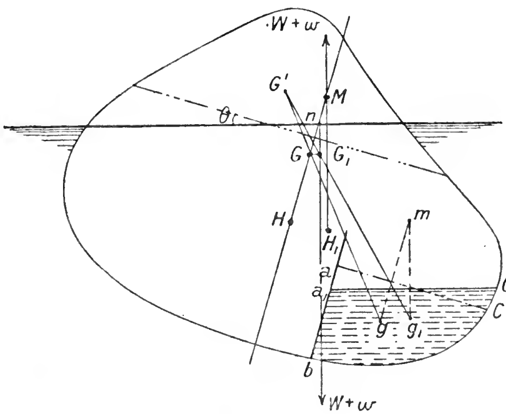


Fig. 20A.

Fig. 20A, such that the position of g changes to g_1, G to G_1, and H to H_1.

Let the vertical lines through G_1 and H_1 intersect HG or HG produced in n and M respectively, and draw gm parallel to HG to intersect the vertical through g_1 in m.

Let ρ and ρ_1 denote the specific gravity of the external and contained fluid respectively.

Then for stable equilibrium

$$(W + w)\,nM = (W + w)\,(GM - Gn)$$

must be positive or M must be situated above n in the line HG.

Now comparing the similar triangles GnG_1 and gmg_1 we have :—

$$\frac{Gn}{gm} = \frac{GG_1}{gg_1} = \frac{GG^1}{gG^1} = \frac{w}{W + w}.$$

$$\text{Therefore } Gn = \frac{w}{W + w} \cdot gm,$$

$$\text{but } \frac{w}{W + w} = \frac{v.\rho_1}{V.\rho} \text{ and } gm = \frac{I_1}{v},^*$$

where v and V are the volumes of contained fluid and fluid displaced respectively, and I_1 is the moment of inertia of free water surface of the contained fluid about an axis through the centre of area of that surface perpendicular to the plane of rotation.

$$\text{Therefore } Gn = \frac{I_1}{V} \cdot \frac{\rho_1}{\rho},$$

and the above expression becomes

$$(W + w)\left(GM - \frac{I_1}{V} \cdot \frac{\rho_1}{\rho}\right),$$

$$\text{or since } GM = HM - HG = \frac{I}{V} - HG,$$

the condition for stable equilibrium is that

$$(W + w)\left(\frac{I - I_1 \cdot \frac{\rho_1}{\rho}}{V} - HG\right) \text{ is positive.}$$

If the contained fluid were water or of the same density as that of the fluid in which the vessel floats, then $\rho_1 = \rho$, and the above expression becomes :—

$$(W + w)\left(\frac{I - I_1}{V} - HG\right),$$

and the restorative moment for a small inclination θ is

$$(W + w)\left(\frac{I - I_1}{V} - HG\right)\theta.$$

* The expression $gm = \frac{I_1}{v}$ requires a passing explanation. Referring to Figs. 20B and 20C, it is clear that if instead of the vessel floating at the water line *ac* we were to consider it only as containing fluid to the same level, the centre of gravity "*g*" of the contained fluid would be identical with H for all inclinations of the body, and therefore *gm* would be identical with HM. Therefore by analogy the expression for *gm* (see Fig. 20A) would be identical with that obtained by considering the compartment *abc* as detached from the vessel and floating at the water level *ac* without the contained fluid, *i.e.*, $gm = \frac{I_1}{v}$ for a small inclination.

Comparing this with the expression for restorative moment given on p. 26 for the case of a vessel with no free fluid, we see that the effect of any contained fluid which is free to move is to reduce the effective metacentric height by the amount $\frac{I_1}{V}$.

The expression $\left(\frac{I - I_1}{V} - HG\right)$ is termed the "virtual" metacentric height of the vessel.

If the angle of inclination be considerable M is no longer the metacentre, and GM represents when multiplied by sin θ the

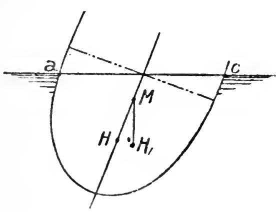


FIG. 20B.

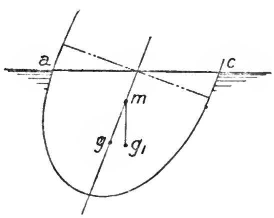


FIG. 20C.

horizontal position of H relative to G, and the restorative moment becomes :—

$$(W + w)\left(GM - \frac{w}{W + w} \cdot gm\right) \sin \theta,$$

where gm-sin θ is the horizontal movement of the centre of gravity of the contained fluid.

That there is an advantage in the case of a vessel carrying liquid cargo or freight in so confining the liquid that motion of the latter is prevented or at least limited as the vessel heels is apparent from these formulæ. In practice, where liquids are carried either in bulk or in comparatively small quantities, it is therefore a fundamental principle that the compartments containing the liquid be either completely filled or be so constructed that in the event of their being only partially filled the contained liquid will have a restricted flow.

It was stated on p. 25 that the point M, Fig. 19, at which for small angles of inclination the line of action of the upward pressure of the water through the centre of buoyancy intersects the initial vertical plane through H and G is termed the metacentre. It has been further shown that the normal

position of the floating body is one of stable equilibrium only so long as the position of G, the centre of gravity, does not rise above M, the metacentre. Hence the term "meta" or "limit" centre. Again, M has been defined geometrically as the "centre of curvature of the curve of buoyancy at H." If now we incline the body through a considerable angle, such that H_1, Fig. 21, is the centre of buoyancy, we find that the centre of curvature of the curve of buoyancy at this point will generally not lie in HG, but on one side or the other of it. Such points, other than M, representing the centres of curvatures of the curve of buoyancy have been termed "pro-metacentres"; the locus of all such points, including the metacentre M is the

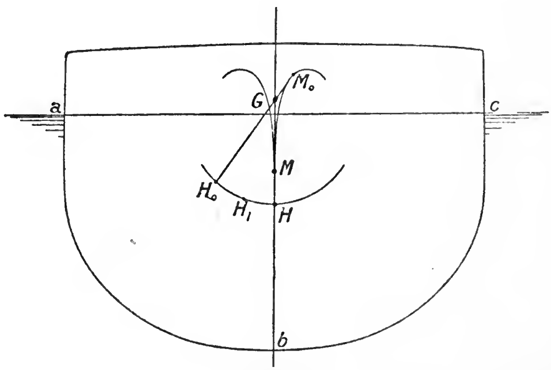


FIG. 21.

"evolute of the curve of buoyancy," and is such that if at any point on the evolute a tangent be drawn to it to intersect the curve of buoyancy the tangent will be normal to the latter at the point of intersection. If the body be now inclined until this normal becomes vertical, the angle contained by the latter and HG is the angle of inclination of the body, and its perpendicular distance from the centre of gravity G is the righting arm for statical stability at that angle of inclination.

There will be a break in continuity of the "evolute of the curve of buoyancy" at the point M in the case where the vertical plane through HG perpendicular to the plane of displacement *abc* divides the body symmetrically, and the curve will be divided into two symmetrical branches having a cusp at M, and the curve of buoyancy will also be divided

symmetrically. If the curve of buoyancy be such that H is a point of maximum or minimum curvature, the cusp of the evolute will point downwards or upwards, as the case may be. This is clearly indicated in Figs. 21 and 22 respectively, in which it will be seen that for inclination in any specific direction—say anti-clockwise, that branch of the evolute of the curve of buoyancy corresponding to H_1, the centre of buoyancy—will in the one case be on the opposite side, and in the other on the same side of HG to the centre of buoyancy.

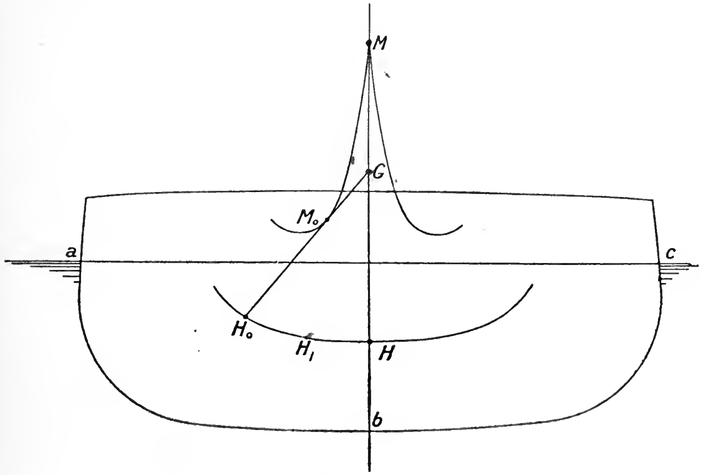


Fig. 22.

Figs. 21 and 22 represent two specific types of floating body, whereof the latter may be described as being of much shallower draught or of much less depth below water as compared with the former.

If the angle at which the body is inclined be such that the tangent to the evolute passes through the centre of gravity G, the righting arm becomes zero. This is shown in the above figures, where H_0GM_0 and H_0M_0G respectively are two such tangents drawn through G to touch the evolutes at M_0 and cutting the curves of buoyancy at H_0, in each case M_0 is the metacentre corresponding to the centre of buoyancy H_0.

The equilibrium will be stable if M_0 lies above G, as in Fig. 21, and unstable if M_0 lies below G, as in Fig. 22, where H_0G is now vertical, and the angle of inclination at which the righting arm

becomes zero is H_0GH. These figures illustrate the law already referred to on p. 19, that the positions of stable and unstable equilibrium follow each other alternately; for whereas in Fig. 21 the normal position is one of unstable equilibrium, the second position in which H_0GM_0 is vertical becomes one of stable equilibrium; and in Fig. 22 the normal position is one of stable equilibrium, while the second position in which H_0M_0G becomes vertical, is one of unstable equilibrium.

The geometrical relation between the curve of buoyancy and its evolute in the case of a body the principal section of which is an equilateral triangle is illustrated in Fig. 23, the condition of flotation being the same as that cited on p. 17. The curve HH......, as before, shows the path of the centre of buoyancy, and the curve MM...... its evolute, or, more correctly, the evolute of that portion of the curve of buoyancy marked from 1 to 5, that is, for a rotation of the body through 120°. This portion only of the evolute has been drawn for the sake of clearness, but the reader may complete it, on a somewhat larger scale, for a further rotation to 240°, and thence to 360°—a complete rotation of the body. It would then be found that the curve as shown in the figure would be twice repeated, although naturally differently disposed. The points marked 1, 2, 3, etc., on the evolute correspond with those similarly marked on the curve of buoyancy.

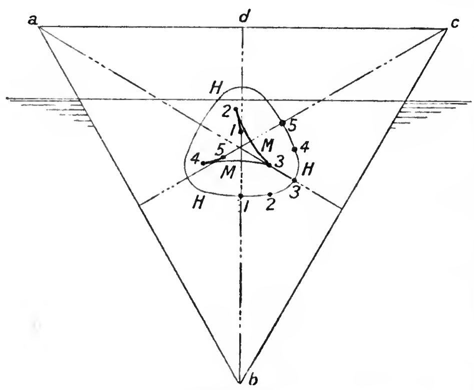


FIG. 23.

Starting from the normal position of equilibrium indicated in the figure, we will gradually incline the body through 120° in the clockwise direction.

The points 1, 3, and 5 on the buoyancy curve are positions of maximum curvature, of which the curvature at 3 is greater than that at 1 and 5, whereas the points 2 and 4—corresponding to the positions of the body in which *c* and *a* respectively

are situated in the waterplane—are points of minimum curvature.

Turning now to the evolute of the curve of buoyancy, we find, starting from M_1, the metacentre for the normal condition, that as the body inclines the curve rises to M_2, situated a little to the left of *bd* when the angle *c* reaches the water surface, thereafter falling to M_3 when *bc* becomes horizontal, again rising to M_4 and falling to M_5 respectively as *b* reaches the water surface and *ab* assumes the horizontal above the water surface.

As seen, the cusps at M_1, M_3 and M_5, are directed towards the points of maximum curvature H_1, H_3, and H_5 respectively on the buoyancy curve, whereas the cusps at M_2 and M_4 recede

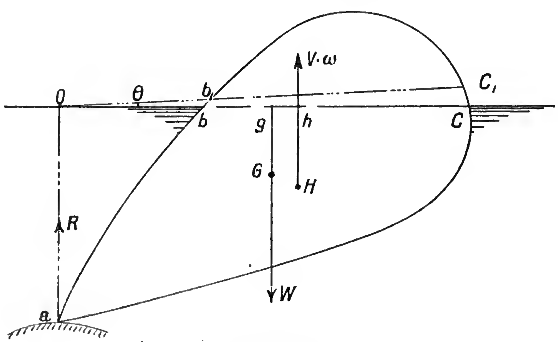


FIG. 24.

from the positions of minimum curvature H_2 and H_4. The pronounced character of the cusp at M_3 agrees with the point 3 on the buoyancy curve, being one of the three positions of absolute minimum curvature.

Constraint.—The stability of a body floating under constraint will now be concisely considered. The subject was referred to on p. 4, where it was shown that in all cases of constraint the constraining force was vertical and in value equal to the difference between the weight of the body and that of the fluid displaced.

In general, if such a body be displaced from its position of equilibrium about the point or axis at which it is supported, even a small angular disturbance of the body will produce an appreciable change in the volume of displacement.

If, however, the axis of constraint be vertically above or below the centre of gravity of the plane of flotation, the change in the volume of displaced fluid will for a small angular movement be negligible and the question of stability is easily determined. This, however, is only a particular instance of the general case, which latter will now be considered, leaving the student to particularise in other specific cases.

Consider the body *abc*, Fig. 24, floating in equilibrium under constraint to receive a small angular movement $d\theta$ about the point or axis of constraint at *a*.

Let *o*, *g*, and *h* be the points of intersection with the water surface of the lines of action of R, the constraining force at *a*; W, the weight of the body; and $w \cdot \mathrm{V}$, the weight of fluid displaced before the equilibrium is disturbed.

Through *o* draw ob_1c_1 inclined at an angle $d\theta$ to the water plane *abc*.

For a small rotation of the whole figure about *a* it is evident that the point *o* will depart from the horizontal *obc* a distance which is negligible, and the new water plane or plane of flotation may be considered as passing through *o* without appreciable error; hence b_1c_1 may be considered as the position of the new plane of flotation in the body.

The loss of moment about *o* due to the change of position of H is $= w \cdot \mathrm{V} \cdot \mathrm{H}h \cdot d\theta$.

Restorative moment due to the increase of fluid displaced, viz., the volume b_1bcc_1, is

$$= \int_b^c \left(dy \cdot x \cdot \frac{x \cdot d\theta}{2}\right) \frac{2x}{3} \cdot w = wd\theta \int_b^c \frac{x^3}{3} \cdot dy = w \cdot \mathrm{I} \cdot d\theta,$$

where I = moment of inertia of plane of flotation *bc* about the axis of *y* at *o*.

$\therefore$ Resultant restorative moment $= w \cdot \mathrm{I} \cdot d\theta - w \cdot \mathrm{V} \cdot \mathrm{H}h \cdot d\theta$

$$= wd\theta(\mathrm{I} - \mathrm{V} \cdot \mathrm{H}h).$$

Now the loss of moment of W about *o* is :—

$$\mathrm{W} \cdot \mathrm{G}g \cdot d\theta.$$

Therefore for stable equilibrium we must have :—

$$w \cdot d\theta(\mathrm{I} - \mathrm{V} \cdot \mathrm{H}h) + \mathrm{W} \cdot \mathrm{G}g \cdot d\theta \text{ is positive,}$$

$$i.e., \; w(\mathrm{I} - \mathrm{V} \cdot \mathrm{H}h) + \mathrm{W} \cdot \mathrm{G}g \text{ is positive}$$

with the condition also that

$$\mathrm{W} \cdot og - w \cdot \mathrm{V} \cdot oh = o.$$

In concluding the remarks in this chapter brief reference will now be made to the subject of oscillation and of the axis about which such oscillation takes place in the case of a body, floating initially in a position of stable equilibrium, experiencing angular displacement.

The determination of the position of the axis about which a floating body is at any instant rotating may generally be stated to be incapable of exact solution. In addition to the weight of the body and that of the fluid displaced by it—two vertical forces—we have, as soon as motion ensues, forces due to fluid friction, direct resistance of the water to the motion of the body, resistance due to the creation of waves, and the effective forces in virtue of the motion of the body itself, brought into play. Further, an arbitrary displacement of the body would in addition to causing angular displacement generally involve vertical and horizontal changes in its position and add to the difficulty of the problem; consequently we are compelled to consider the movement under certain assumed conditions.

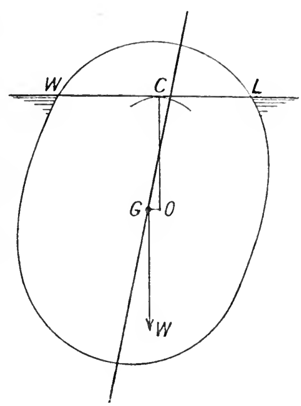


FIG. 25.

It is usual to assume that the volume of water displaced by the body during its motion remains constant and the same as when the body is floating in its normal position of rest; this is only approximately correct, due to the action of the effective and impressed forces.

First consider the body floating in still water, that the water is a perfect fluid offering no resistance to its motion, and that as the body is inclined from the normal position the volume of displaced fluid remains constant; under such a condition the water surface will be always tangential to the curve or surface of flotation.

The case resembles that considered on pp. 13, 14, except that there we assumed the ideal condition of the surface of flotation and the water plane being always in contact, without considering the effect of the acting forces.

Let c be the point of contact of the surface of flotation and

water plane for the position of the body indicated in Fig. 25; for any further angular movement the point *c* will move horizontally in the plane of the water surface WL, and therefore the axis of rotation will have its position in the vertical line (or plane) *co* perpendicular to the water plane. The only forces influencing the motion of the body are the weight and buoyancy, both of which act vertically; therefore the centre of gravity G will move vertically, and the instantaneous axis about which G will rotate lies in the horizontal through G, *i.e.*, in G*o*.

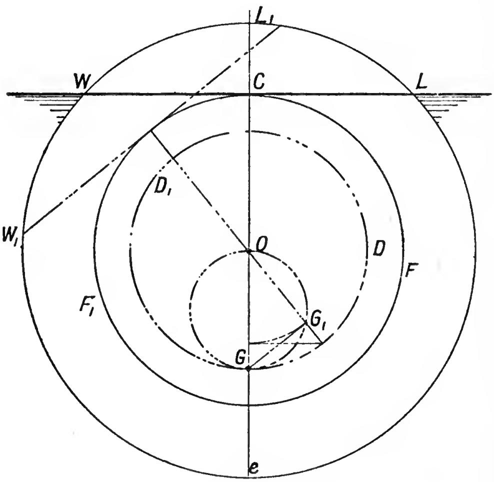


Fig. 26.

The point *o*, in which G*o* intersects *co*, is the position of the instantaneous axis of rotation of the body for this position.

For other angular displacements of the body the point *c* will take up a varying position along WL, while G moves in the vertical GW, so that the position of the axis of rotation *o* will vary.

It will be seen that for the position wherein the line joining the centre of gravity of the body and the centroid of the plane of flotation is vertical the axis of rotation will initially coincide with the position of the centre of gravity.

The case of a body of circular cylindrical section is instructive and is illustrated in Fig. 26, which shows the body floating in its normal position of equilibrium.

The locus of the instantaneous axis of rotation relative to the body is obviously the circle OGG_1 described on OG as diameter, where O is the centre of the section and G the centre of gravity of the body. The absolute locus in space is the circle GDD_1 of radius OG, which circle is seen to envelop the locus OGG_1.

As the body inclines it moves in the horizontal direction, and

this translatory movement is measured by OD, the amount by which the centre O is displaced during an angular displacement $\frac{\pi}{2}$ of the body; the whole movement for an angular displacement of 2π being a harmonic oscillation relative to the vertical plane COG, the amplitude of the oscillation being the radius OG of the circle GDD_1.

The case of a body floating in a perfect fluid and experiencing no resistance to its motion is entirely hypothetical. In practice the conditions would be quite different to those outlined above. Apart from the fact that we would be dealing with an imperfect fluid, the hypothesis of no resistance would not be nearly realised. The above problem, however, is both interesting and instructive, will repay careful study, and will be found helpful in approaching the subject of oscillation as it is actually presented to us in practice.

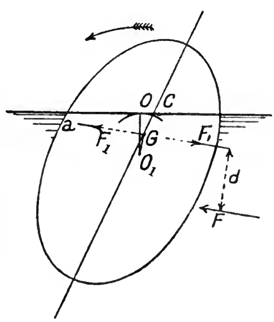


FIG. 27.

If we introduce the condition that the body experiences resistance to its motion, the effect will be to displace the centre of gravity from the vertical *co*G, either in the one direction or the other, according to the angular position of the body and the direction of rotation; and the position of the instantaneous axis, and the locus of same, will be modified. The exact position of the axis, except in certain specific positions, is indeterminate, and all that will be attempted here is to indicate the nature of the variation of the position as the body becomes inclined.

When a body moves through a fluid, whether in a straight line or a curvilinear path, the surrounding fluid is affected by and moves with it; the moving fluid may be divided into that due to stream line motion and the drag water moving behind in the wake of the body. Any acceleration given to the body is attended by an acceleration of the fluid, and the mass of the body is *virtually* increased.

If we could determine the increase of mass and the change in velocity, the external forces acting on the body could be determined. This is the difficulty presented in the case of a

body floating in a fluid and subject to angular displacement. At the beginning of the movement, when the body is at its extreme angle of inclination, its velocity is zero, increases as the angle of inclination diminishes, reaching a maximum with acceleration zero at the normal or upright position, and diminishing again to zero at the extreme angle of inclination on the other side.

At the extreme inclination, when the body is just assuming a momentary position of rest, the effective forces are *nil*, and the impressed forces balance, so that the body is free to move about an axis through its centre of gravity. As the body returns to the normal position the fluid effect is to oppose its motion; suppose F to represent the resultant effect due to the resistance of the fluid, and let two equal and opposite forces F_1 equal to F be applied at G, the centre of gravity of the body, their resultant effect on the system will be *nil*.

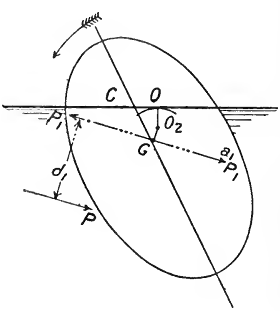


Fig. 28.

Combining them with F we have a couple $F \times d$ tending to check the angular movement of the body, and a force F_1 (= F) acting in the direction G*a* and tending to move the centre of gravity G in that direction. The axis of rotation will therefore be situated below G, as at o_1, in Fig. 27. When the upright position is reached the velocity of rotation reaches its maximum value, the acceleration is zero and the resultant force acting on the body is zero, or the axis will again be situated at G.

As the upright position is passed and the body assumes inclination on the other side the energy of the fluid already acquired will assist the angular motion of the body. Suppose P now to represent the resultant fluid action, then imposing two equal and opposite forces P_1 (= P) at G and combining them with P, we have a couple $P \times d_1$ assisting the angular movement of the body and a force P_1 tending to move the centre of gravity G, and therefore the body, in the direction Ga_1. The axis of rotation will therefore be situated above G, as shown at o_2 in Fig. 28. When the extreme angle of inclination

is reached the resultant effective force will again be zero, and the axis will again be situated at G.

It will be observed that the force F or P, representing the action of the fluid, apart from that of buoyancy, is variable, increasing or diminishing in value and changing in direction according to the inclination and dependent upon the angular velocity of the body. The virtual mass of the body will depend upon this latter, and therefore the centre of gravity G will vary in position relative to the body, according to the angle of inclination and angular velocity acquired, so that G is not necessarily the position of the centre of gravity of the body.

If we consider the movement of the body for each quarter of an oscillation it may be taken that the approximate path of the axis relative to the body is a closed curve, and for the complete oscillation four such curves crossing each other at some point near the centre of gravity of the body; similarly, the locus in space would be other four curves, crossing at the same point, and each enveloping or containing the corresponding curve representing the path of the axis relative to the body.* The condition would be somewhat analogous to that shown to exist in the case of the cylinder on p. 38, in which the path of the axis in space, viz., the circle GDD_1, enveloped the locus of the axis relative to the body, viz., the circle OGG_1, these circles being in contact at G, the centre of gravity of the body.

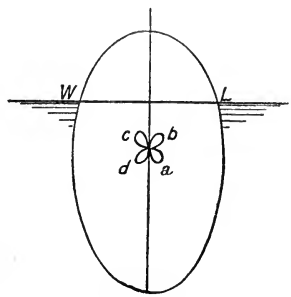


FIG. 28A.

In the general case the locus of the axis relative to the body would be somewhat as indicated by *abcd* in Fig. 28A, in which the branches *a* and *b* respectively correspond to the conditions represented in Figs. 27 and 28.

It is evident that generally the volume of fluid displaced will vary according to the direction of the resultant of the fluid forces at any particular instant, and the path of the axis of rotation will in consequence be affected.

Oscillation.—The oscillation of a floating body may be

* See also a paper on "The Accelerated Motion of Bodies in Water," by A. W. Johns, Trans. I. N. A., 1909.

brought about by a disturbance of equilibrium in such a manner that the movement takes place entirely in the vertical direction, in virtue of an increase or diminution in the volume of water displaced; or, assuming the displacement to remain constant, the body is inclined through some external cause from its initial position of rest.

Both vertical and angular oscillation would, however, generally result from an arbitrary disturbance of equilibrium, and, although the two oscillations are not independent, we may still determine the equations relating to these motions.

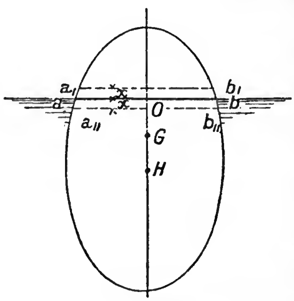


Fig. 29.

We will first consider a disturbance of equilibrium in the vertical direction only, and assume that the surface of the fluid in which the body floats is so large compared with the horizontal section of the body that its level is not affected by the motion.

Assume that the vertical through H and G intersects the water plane in its centre of gravity or area o, and that the form of the body is such that during the oscillation the planes of flotation are parallel.

Let the body be depressed initially through a distance a, Fig. 29, and then released. It will oscillate above and below the fluid surface, and the effective force acting on the body is

$$M\frac{d^2x}{dt^2},$$

where M is the mass of the body and x the vertical displacement at any instant during the motion.

The impressed force is $-A \cdot x \cdot w$, *i.e.*, the increase or diminution in weight of water displaced.

$$\text{Hence } M\frac{d^2x}{dt^2} = -A\,.\,x\,.\,w$$

$$\text{or } \frac{d^2x}{dt^2} = -\frac{A\,.\,x\,.\,w}{M} = -\frac{A\,.\,g}{V}\,.\,x,$$

which, integrated between the limits a and $-a$, gives the time of a complete oscillation—

$$T = 2\pi\sqrt{\frac{V}{g\,.\,A}},$$

when V = volume of displacement, and A = area of plane of flotation.

If HG produced had not intersected the plane of flotation in its centre of gravity, then for a vertical displacement of the body the additional or reduced weight of water displaced would not have its centre of gravity in the line HG, *i.e.*, the centre of buoyancy for the new displacement would move out of the initially vertical line HG, with the result that there would be a statical moment causing angular movement, the moment being $(W \pm w)z \cdot \theta$, where

w = increase or decrease of water displaced.
z = metacentric height.
θ = angle of inclination, which is considered small.

So long as x is small, θ is small, and the angular movement may be neglected; but as soon as x assumes a large value the angular movement θ would have to be considered, and the above formula, including the metacentric height z, would not hold.

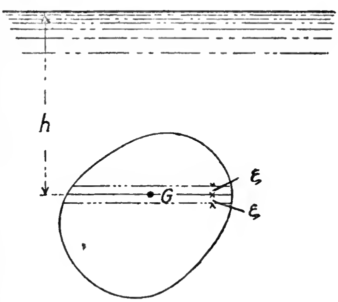


FIG. 30.

Before considering angular oscillation, however, we will take the interesting case of a homogeneous body floating in equilibrium completely immersed in a fluid the density of which varies with the depth below the fluid surface.

Let the centre of gravity G of the body be initially situated at a distance h below the fluid surface, Fig. 30.

Consider an elementary volume dv at a depth x, and let $k.x$ denote the density of the fluid at this point, where k is a co-efficient.

$$\text{Mass of fluid displaced} = \int dv \,.\, k.x = k \,.\, V \,.\, h,$$

where V is the volume of the body, or

Mass of body = mass of fluid displaced.
= volume of body × density of fluid at the depth of its centre of gravity.

Now suppose the body to suffer a vertical displacement ζ and then released, we have as before :—

$$M\frac{d^2\xi}{dt^2} = \text{change of displacement}$$
$$= \text{weight of body} - \text{weight of fluid displacement}$$
$$= g \,.\, V \,.\, k \,.\, h - g \,.\, V \,.\, k\,(h + \xi) = -g \,.\, V \,.\, k \,.\, \xi,$$

$$i.e., \; V \,.\, k.h\frac{d^2\xi}{dt^2} = -g \,.\, V \,.\, k \,.\, \xi,$$

$$i.e., \; \frac{d^2\xi}{dt^2} = -\frac{g}{h} \,.\, \xi$$

$$\text{or } T = 2\pi\sqrt{\frac{h}{g}}$$
$$= \text{time of a complete small oscillation.}$$

The third case to be considered is that of a body floating at rest at the surface of the fluid and suffering a small angular displacement in such a manner that the volume of fluid displaced remains unaltered. Assume, as before, that HG produced passes through O, the centre of area of the plane of flotation; under such a condition it will be immaterial, so far as the motion is concerned, whether the oscillation be considered about O or G—for small inclinations (Fig. 31).

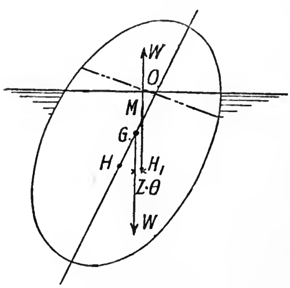


Fig. 31.

For an angle of inclination θ the restorative couple is $W{\cdot}z{\cdot}\theta$, where z = metacentric height.

Let ρ = radius of gyration of the vessel relative to the axis at G. Then, equating the moments of the effective and impressed forces, we have :—

$$M\rho^2\frac{d^2\theta}{dt^2} = -M \,.\, g \,.\, z \,.\, \theta$$

$$\text{or } \frac{d^2\theta}{dt^2} = -\frac{g \,.\, z}{\rho^2} \,.\, \theta.$$

Integrating :—

$$\left.\frac{d\theta}{dt}\right|^2 = \frac{g \,.\, z}{\rho^2}(\varphi^2 - \theta^2),$$

where φ = limiting angle of oscillation.

$$\therefore \mathrm{T} = \int dt = \sqrt{\frac{\rho^2}{g \cdot z}} \int \frac{d\theta}{\sqrt{\varphi^2 - \theta^2}}.$$

$$= \sqrt{\frac{\rho^2}{g \cdot z}} \Big(\sin^{-1}\frac{\theta}{\varphi}\Big)_{-\varphi}^{\varphi}$$

$$= \pi\sqrt{\frac{\rho^2}{g \cdot z}} = \begin{array}{c}\text{time of a half oscillation (swing}\\ \text{from side to side).}\end{array}$$

If the line or plane HG does not pass through *o*, then for an arbitrary or angular displacement the volume of fluid displaced is not constant and the vertical and angular oscillations cannot be separated. The solution leads to equations more complicated than those given above.

CHAPTER II

SHIPS

In Chapter I. we have explained the fundamental principles of the action of fluids in relation to the stability of floating bodies in general ; it now remains to apply those principles to bodies of specific shape or form.

The present chapter will be devoted to ordinary ship-shape bodies, to which the principles of stability have a direct application of great practical value ; before entering upon a detailed discussion of these points, however, we will consider in a general manner the *importance of the study of stability* and how it is enhanced by the fact of the possible variation of the governing factors, especially with reference to the constituents which go to make up the *whole* of the weight of a vessel and its freight.

Essential features in the design of a vessel for seagoing purposes are strength and stability. Whatever the purpose for which the vessel is to be built, these two characteristics form the primary features of the design.

There are, naturally, other factors upon which the success and efficiency of the vessel will depend ; the question of speed and, if the vessel be of the passenger class, comfort of the human element are points of considerable moment to the owner. Upon the one will depend very largely the earning power of the vessel, and upon the other the attractiveness to those whose business or pleasure, as the case may be, leads them to seek the vessel most likely to meet their requirements.

Whatever the relative importance of these several factors may be, stability stands out as being essential to the well-being and efficiency of the vessel. It is determined not only by the form, but also by the amount and distribution of the weight of the structure as a whole, and of such varying quantities as fuel, cargo and ballast ; and generally no change, whether in respect of quantity or position, may be made in any one of these items without affecting the stability.

No absolute standard can be set up by which we may assign theoretically a definite desirable amount of stability irrespective of the type or class of vessel under consideration ; rather the designer aims at giving such an amount as experience teaches him to be necessary for the well-being of the particular type of vessel he has in view. Here then at least is an instance where the blending of experience and fundamental scientific principles is required.

Once the question of stability has been decided upon it is essential that the amount should neither be diminished nor increased—at least within reasonable yet small limits—as the condition of the vessel changes on account of the discharge or loading of cargo, the burning of fuel, etc. Negligence in this respect would be likely to cause excessive inclination, uneasy rolling, and general discomfort when the vessel is called upon to meet exigencies of a trying nature at sea.

It is as important to guard against excessive stability as it is necessary to see that the stability is not reduced below a minimum amount, the limiting values being dependent upon the size and type of vessel. Each case has its particular evil ; on the one hand, uneasy rolling may cause or tend to cause straining of the structure, and displacement of fittings, and of those portions of the cargo or freight which may be free to move, as, *e.g.*, grain, coal, etc. ; on the other hand, excessive inclination may result in a diminished power of the vessel to right herself.

It has been adequately shown in the preceding chapter that stability is governed by the relative position of the centre of gravity—a fixed point for any particular condition of loading, and of that point at which the vertical line through the centre of buoyancy when the vessel is inclined intersects the initial vertical plane containing the centre of gravity and centre of buoyancy when the vessel is floating in the normal or upright position.

The greater the distance between this point of intersection and the centre of gravity, the greater will be the righting power ; and the less the distance, the smaller the righting power of the vessel when inclined.

For small inclinations this point is practically fixed, but for inclinations which may be described as more than small, *i.e.*,

exceeding about 8° or 9° on either side of the upright position, the point is variable in position and the designation "shifting metacentre,"* has been given to it. It is related to what in Chapter I. has been termed the "pro-metacentre," inasmuch as it is the point where the initial vertical plane through the centre of gravity and the centre of buoyancy is intersected by the normal to the curve (or surface) of buoyancy at the centre of buoyancy for the particular inclination.

This is shown more explicitly in Fig. 32, in which HH_1 represents the curve of buoyancy, H_1 being the centre of buoyancy for the inclined position, and $H_1M^1m_1$ the radius of curvature of the curve of buoyancy at H_1; m_1, the centre of curvature, is the "pro-metacentre" for this particular inclination of the vessel, and M^1 the point referred to above, in which the normal H_1m_1 to the curve of buoyancy intersects the initial vertical plane HG.

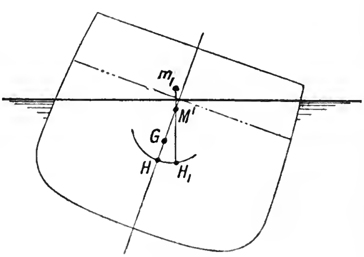


Fig. 32.

It is evident that for any definite inclination of the vessel the righting arm, and therefore the righting power or movement, will depend upon the value of GM^1. For small inclinations up to 8° or 9° the point M^1 is practically fixed, is the centre of curvature of the curve of buoyancy at H, and is termed the "metacentre."

For the vessel to possess the power to right herself when inclined, it is plain that the centre of gravity G must fall below M^1 in the plane HGM^1; hence the designer, in order to satisfy this condition, has to mould carefully the shape or form of the vessel, at the same time aiming at a proper distribution of weight in the structure and equipment.

The distribution of weight is naturally determined by the conditions to be fulfilled in the design, and will depend upon

* The term "shifting metacentre" can hardly be considered a happy expression. The point so designated is really not a metacentre at all, as it does not refer to the upright position of the ship, in which the term "metacentre" found origin. In France, however, the term has been more freely used, and the point has also been termed the "metacentre at the angle θ," where θ denotes the inclination of the vessel. The point evidently marks the limiting position on the middle line above which the centre of gravity may not be raised without causing the vessel to incline beyond the angle θ.

the specific purpose for which the vessel is to be built. In the case of war-vessels the problem is somewhat simplified, inasmuch as at the outset the exact requirements are known; and, further, the condition of the vessel when built is practically unalterable except for the comparatively slight modifications in draught and position of centre of gravity brought about by variations in the amount of stores, fuel, ammunition, etc.

With the mercantile vessel, and more especially with that type primarily used for carrying cargoes, there are greater difficulties to be faced, and for the reason that there are so many fluctuating quantities and conflicting claims which have to be met in the finished design. Apart from fuel and stores, we have to take into consideration the effect resulting from a continually changing cargo as the vessel proceeds from port to port. On one voyage a homogeneous cargo might be carried, as, for example, cotton bales, of so light a weight, or of so little density, that the whole cubic content of the holds might be occupied without the vessel floating even at a reasonably good draught for seagoing purposes. In such a case water ballast would probably be resorted to in order to bring about not only the desired immersion, but to modify the position of the centre of gravity of the loaded ship, which, due to the light, homogeneous cargo, is possibly situated too high.

After discharging her light cargo the vessel might be required to sail with a heavy freight, such, for example, as iron ore, rails, etc. Such heavy concentrated weights would generally bring the vessel to her load draught to the exclusion of water ballast, and if placed low down would have great influence upon the position of the centre of gravity; so that the distance between this point and the metacentre would now be much greater than in the case with the light cargo of cotton.

Turning from cargo carriers to those vessels primarily intended for passenger service, often with limited cargo-carrying capacity, new conditions arise which require careful consideration.

In this class of vessel designed for swift passage, the machinery and fuel or bunker space occupies most of the volume which ordinarily would be devoted to the stowage of cargo; above this is the passenger accommodation, confined to the 'tween

decks or extending higher, according to the number and class of passengers to be carried.

With the best class of vessel in recent years there has been a tendency to build the accommodation as high in the ship as possible ; this tendency has not found its origin with the designer, nor has it been his wish to increase, often to huge proportions, the height of the above-water portion of the vessel.

On the contrary, it has developed from the desire of passengers to secure greater convenience and better ventilation than is ordinarily obtainable in the comparatively confined 'tween decks.

The demand for better accommodation has been met on the one hand by the shipowner, and on the other by the naval architect, who has been compelled to add to the upper works or superstructures of the modern passenger vessel, until the height above water to which the structure extends may now be found to exceed the mean draught at which the vessel floats ; weight has been added at a position which, generally speaking, is the least desirable, and for compensation —at least in part—there has been a complete abandonment of heavy spars and fittings which one was so accustomed to associate with the earlier steam vessels and sailing ships, but which in a modern liner would be of little or no service.

One result of the development of the superstructure or top hamper—to use a hackneyed expression—is naturally to increase the height of the centre of gravity of the vessel.

But side by side with the growth of the upper works there has been a modification in the principal dimensions, and therefore of the submerged portion of the vessel—the primary factor affecting stability—in order to ensure safety and efficiency.

To the non-technical mind superstructures which bulk largely to the eye almost invariably convey the impression of instability and danger.

The naval architect, however, is not guided by appearance, but by his knowledge of the combined effect of form and the general distribution of weight in the ship.

It is quite possible that a vessel of this type would actually possess greater stability than another of less bulky appearance above water.

A vessel possessing top hamper is not necessarily void of stability any more than one of ordinary type would be necessarily stable. Stability, indeed, depends upon a variety of factors—the position of the centre of gravity (a fixed point for any given condition), the metacentre (a point the position of which is variable with draught), the general dimensions of the vessel, and, with relation to this latter, freeboard and beam, are all very important as affecting stability. All these factors taken collectively, and not any one isolated feature of the design, will determine stability.

This cannot be too well observed by the student in naval architecture, otherwise he may find that the whole problem of a design has to be reviewed through giving insufficient attention to some one feature of seeming comparative insignificance, but which in reality of itself forms a unit of vital importance in the finished design.

The problem of stability, though of itself presenting no great difficulty when carefully approached, yet demands the exercising of a wise discretion in working out the design ; and in one particular does this specially apply, viz., in the distribution of the weights which collectively make up the weight of the finished structure. At this point almost of necessity the work of the designer ends. Beyond this there is always the question of loading, over which he can exercise no power ; all that he can do is to satisfy himself that under certain ideal and specific conditions of loading the vessel would possess good stability qualities, leaving it to the mature judgment of those into whose hands the work of loading may be placed to suitably distribute the freight when these ideal conditions are departed from. It is for this reason that the designer, foreseeing the possibility of an improper distribution of cargo, often allows for such a contingency by designing the vessel with rather more stability in the light condition than he otherwise would.

This naturally applies specially to cargo-carrying vessels. In the case of passenger ships, yachts, and war-vessels, where all or most of the variable elements of the load are known, the designer is not so handicapped.

The distribution of weight besides determining the stability value of the vessel will affect the latter in another way. The same value of stability—or, in other words, the same position

of the centre of gravity—for the same displacement, may be arrived at by various methods of loading, with heavy or light, homogeneous or heterogeneous, cargoes; but it does not follow that the vessel under such distinct conditions would behave equally well at sea.

Were the weights winged or generally distributed away from the common centre of gravity the result would be an increase in the value of the radius of gyration of the vessel as compared with a more concentrated position of those weights, and in consequence the period of roll, or the time taken by the vessel to swing from side to side, would be affected. From a well-known principle in rigid dynamics we know that the mechanical moment of the accelerating forces at any instant involves the product of the "moment of inertia" of the oscillating body and the "angular acceleration"; therefore, assuming the external forces causing motion to remain unchanged, the angular acceleration will diminish, *i.e.*, the rolling motion will be slower with an increase in moment of inertia, and *vice versâ*, provided the common centre of gravity of the vessel remains unchanged.

But suppose the position of the centre of gravity has changed producing an alteration in the value of GM or initial stability of the vessel; the period or time of roll will be affected, inasmuch as the latter varies inversely as the square root of the initial stability or metacentric height. This will be seen by reference to Chapter I., p. 45.

The manner of distribution of the component weights in a vessel will therefore not only affect the stability but with it also the rolling motion. It is on this account, where vessels having a small amount of cargo on board, or even no cargo at all, have to proceed to sea in ballast in order to secure proper immersion of the hull and of the propellers, that wing ballast tanks are preferable to double-bottom tanks as conducive to a less exaggerated effect on the stability, while the effect on the moment of inertia would not be widely different in the two cases—dependent, of course, on the weight of water ballast and the position of its centre of gravity in the wings and double-bottom respectively.

While the stability and the behaviour of the vessel at sea will be largely determined by the loading and the manner of the

distribution of the several items of weight contributing to the loading, yet on the other hand it must be remembered that the primary factor determining the stability—the foundation, so to speak, from which the ultimate stability of the laden vessel is determined—is the *form* of the vessel, meaning by form not only the shape of that part of the body which when she is floating upright is submerged, but also the portion of the body contributing to the reserve buoyancy usually above water, but which due to rolling might be submerged. It is this latter portion of the structure which plays such an important part in determining the righting power and range of stability when the vessel becomes appreciably inclined from the upright position of equilibrium.

The actual form given to seagoing vessels in any particular case—*i.e.*, where certain specified conditions of speed, draught, stability, etc., have to be fulfilled—cannot be assumed from any process of pure scientific reasoning, but rather from what experience teaches as being the most suitable for the particular purpose for which the vessel is intended, tempered by our knowledge of the results of scientific research and experiments on bodies of ship-shape form.

It is the purpose of this work to show, given any particular form of vessel, how the stability may be calculated and recorded; what effect any alteration in dimensions—whether in respect of draught, beam, freeboard, etc.—would have on the value of that stability; and, finally, what the ultimate result would be of loading the vessel in any specific manner for seagoing purposes.

It is now customary in most shipbuilding establishments to make a calculation of the vertical position of metacentre "M," for certain specific conditions of loading and draught, by first calculating the vertical position of the centre of buoyancy "H," and then the value of "HM" for each condition, from the formula given in Chapter I., p. 25, viz. :—

$$HM = \frac{\text{Transverse moment of inertia of water plane area}}{\text{Volume of displacement}}.$$

The vertical position of "M" may now be determined relative to some convenient *fixed* point in the vessel. The point usually chosen for this purpose is the underside of keel at amidships, which point is also convenient for setting off the draughts in the diagram. This diagram consists in setting off a scale of

draughts, and at the point on the scale indicating the draught corresponding to the loaded condition for which the position of " M " has been estimated, set out an ordinate—to scale—perpendicular to the line of draughts, representing the height of " M " above the bottom of keel; this is done similarly for other conditions of loading. A fair curve drawn through all such points is the curve of " vertical position of metacentres above bottom of keel," the value of the curve being that for any draught—other than those to which the calculations have been made—the vertical position of the metacentre may be obtained by measuring the ordinate to the curve set out from the point corresponding to that draught.

Conjointly with this calculation an estimate is made of the height of the centre of gravity (G) of the vessel, together with her complete equipment and load, for each specified condition; and these heights are set out as ordinates on the diagram at the points indicating the draughts corresponding to those conditions. By this means the value of the " metacentric height," or " GM," is obtained for each condition.

The whole diagram so constructed is designated the " metacentric diagram," and upon it is usually drawn the curve or locus of height of centre of buoyancy above bottom of keel.

The ordinary calculations relating to " initial stability," or " metacentric height," or " GM "—all equivalent terms—only take note of the vessel for the upright position, and hence the above-water portion of the vessel does not enter into the calculation, except in so far as it affects the position of the centre of gravity. It is only when we come to estimate the statical stability for appreciable and considerable angles of inclination from the upright position that account is taken of the *above-water* portion of the vessel.

To illustrate the method of constructing a metacentric diagram we propose to take the case of a rectangular or box-shaped vessel of the following dimensions:—

Length	300′ 0″
Beam	20′ 0″
Depth	10′ 0″

To simplify the problem we will assume the vessel to float at the draughts 5·0 ft., 6·0 ft., 7·0 ft., and 8·0 ft. under four

separate conditions of loading, such that the position of the centre of gravity of the vessel remains unchanged and situated at 5·0 ft. above the base or bottom of the vessel.

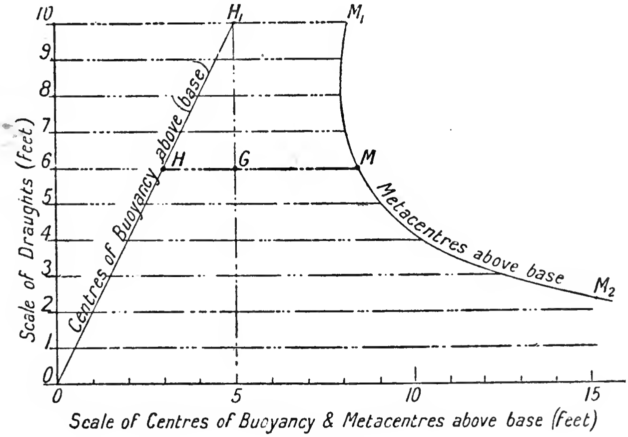


FIG. 33.

The calculation relative to the draught of 6·0 ft. is given below.

(*a*) Centre of buoyancy above base = 3·0 ft.

(*b*) Volume of displacement of vessel = $300 \times 20 \times 6 =$ 36,000 cubic ft.

(*c*) Transverse moment of inertia of water plane area =

$$\frac{1}{12} \times 300 \times 20^3 = 200{,}000.$$

(*d*) Height of metacentre above centre of buoyancy = $\dfrac{200{,}000}{36{,}000} = 5{\cdot}\dot{5}$ ft.,

$$i.e.,\ 5{\cdot}\dot{5} + 3{\cdot}0 = 8{\cdot}\dot{5} \text{ ft. } \textit{above base.}$$

(*e*) Metacentric height or "GM" = $8{\cdot}\dot{5} - 5{\cdot}0 = 3{\cdot}\dot{5}$ ft.

A similar calculation would be made relative to the remaining water lines.

To record the results of our calculation graphically, two methods may be used. These are illustrated in Figs. 33 and 34.

Fig. 33 illustrates the more straightforward method, and is the one usually seen on the "displacement diagram" in private shipyards. From a vertical scale of draughts the heights of

centre of buoyancy and of metacentre above base are set out on horizontal lines at points representing the several draughts; curves drawn fairly through all such corresponding points are the locii or curves of "height of centre of buoyancy" and of "height of metacentre" above base. The diagram may be completed by marking thereon the height of centre of gravity above base at the draughts corresponding to the specified conditions of loading; in this case the height is constant and equal to 5·0 ft., and is indicated by the vertical line drawn through G.

Fig. 34 illustrates the "Admiralty method" of constructing a metacentric diagram, and is usually followed in the case of war-vessels; it is not quite so quickly constructed as that shown in Fig. 33, but when once drawn is just as easily read. From the draught marks set up on a vertical scale horizontal lines are drawn to intersect a straight line passing through the zero draught point and inclined at an angle of 45°. Through these points of intersection vertical lines are drawn, upon which are set up the heights of centre of buoyancy and metacentre above base corresponding to the several draughts, and curves drawn fairly through all such corresponding points as in the first case. The diagram may be completed by marking thereon the positions of the centre of gravity as before.

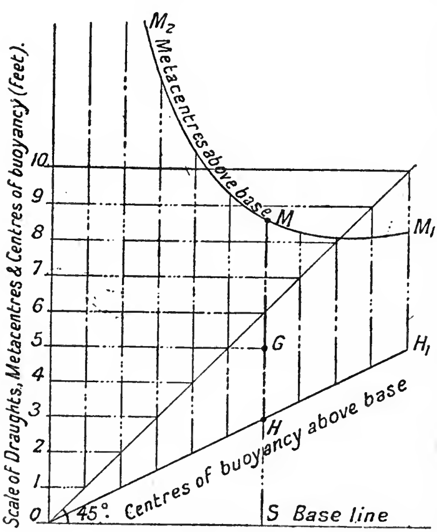


Fig. 34.

The thick broken line HGM in each figure represents the case for the 6·0-ft. draught calculated above.

On that side of the diagram adjacent to the scale of draughts a table is often arranged giving particulars of tons per inch of immersion, and displacement in tons, at several intervals of

draught; this will be referred to later when dealing with the metacentric diagram for an actual ship.

There are one or two points relating to the curve of metacentres in the case of a rectangular-shaped vessel which are both of interest and instructive, the consideration of which will be appreciated when we come to consider vessels of ship-shape form and the metacentric curves relating thereto.

Referring to Fig. 34, let a, b, and c denote the length, beam, and depth respectively of the box-shaped vessel; we have :—

$$\text{Height of centre of buoyancy above base} = \text{SH} = \frac{\text{draught}}{2}$$

$$= \frac{\text{OS}}{2} = \frac{x}{2},$$

where $x = \text{draught} = \text{OS}$.

$$\text{And HM} = \frac{\frac{1}{12} \cdot a \cdot b^3}{a \cdot b \cdot x} = \frac{b^2}{12x}.$$

$$\therefore \text{SM} = \text{SH} + \text{HM} = \frac{x}{2} + \frac{b^2}{12x} = y \text{ (say)},$$

which is the equation to a hyperbola, *i.e.*, the metacentric curve is a hyperbola represented by the equation

$$6x^2 - 12xy + b^2 = 0 \quad . \quad . \quad . \quad . \quad (1)$$

referred to the base line as axis of x, and the line of scale of draughts as axis of y.

$$\text{When } x = o, y = \infty,$$

or the axis of y (vertical line of scale of draughts) is an asymptote to the curve.

Now the equation of the straight line OH is $y = \frac{x}{2}$, which, combined with equation (1), gives $x = \infty$, or the line of centre of buoyancy OH meets the metacentric curve at infinity, *i.e.*, is asymptotic to it.

In reality the curve of metacentres, although proceeding to infinity along the axis of y, will be limited in the direction of the axis of x at that point representing the maximum draught of the vessel, *i.e.*, at which draught = depth of vessel; for if the vessel be *just* submerged the metacentre and centre of buoyancy are coincident, so that the curve suddenly falls from M_1, vertically to H_1, on the line of centre of buoyancy. The

curve of metacentres is therefore $H_1M_1MM_2$, unlimited towards M_2, but limited at M_1—when the vessel is *just* submerged, and falling to H_1—for all submerged conditions of the vessel.

This is also indicated in Fig. 33.

The complete curve, if drawn, would represent the case of a vessel of *infinite* depth compared with the remaining linear dimensions.

The student might with profit investigate for himself the nature of the metacentric curves for other vessels of simple geometric forms, concerning which the limited space at our disposal will not permit of solution here.

It was shown on p. 26 that in the case of a vessel subject to a small inclination from the normal or upright position of equilibrium, *i.e.*, in which the metacentric method of stability applies, the restorative moment is given by

$$W\left(\frac{I}{V} - HG\right)\theta . = W . GM . \theta,$$

$$\text{where } GM = HM - HG = \frac{I}{V} - HG.$$

If, however, the vessel be inclined to some considerable angle φ, the restorative moment, as given on p. 27, becomes

$$W . Gz = W\left(\frac{v . nn_1}{V} - HG \sin\varphi\right) \quad . \quad . \quad \text{(See Fig. 19.)}$$

In any particular case V and HG are calculable quantities, having given the lines and scantlings of the vessel; hence for any given inclination φ, the righting moment resolves itself into the determination of the product $v.nn_1$. The process of finding the value of this product in the case of bodies of irregular or of ship-shape form would, if calculated geometrically, involve an immense amount of work which it is not proposed to enter into here; for information on this point, however, the reader is referred to the treatise on "Stability of Ships," by Sir E. G. Reed.

The calculation is now usually made mechanically by the aid of the "Integrator," an instrument the value of which cannot be overestimated, and the process adopted will be explained later.

Before applying the general formula to ship-shape bodies, it will be instructive to consider the rectangular or box-shaped

vessel, homogeneous if solid, otherwise completely empty or entirely filled with a homogeneous substance, so that however the vessel may move the centre of gravity remains a fixed point within it.

We will incline the vessel by stages through 90° and determine for each inclined position chosen the value of the righting arm Gz, from which to construct a curve of stability. The dimensions of the vessel are the same as those given on p. 54, and we will first consider the draught of 5·0 ft., the centre of gravity being situated 5·0 ft. above base.

The vessel being entirely symmetrical, it will only be

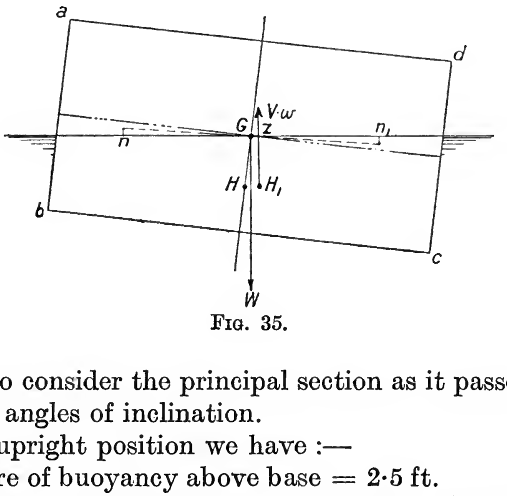


FIG. 35.

necessary to consider the principal section as it passes through the several angles of inclination.

For the upright position we have :—

(*a*) Centre of buoyancy above base = 2·5 ft.

(*b*) Height of metacentre above centre of buoyancy =

$$\frac{\frac{1}{12} \times 300 \times 20^3}{300 \times 20 \times 5} = 6\cdot\dot{6} \text{ ft.}$$

$$i.e., = 6\cdot\dot{6} + 2\cdot5 = 9\cdot1\dot{6} \text{ ft. above base.}$$

Therefore GM $= 9\cdot1\dot{6} - 5\cdot0 = 4\cdot1\dot{6}$ ft.

Next consider the vessel inclined at an angle of 10°, we have, from the general formula :—

$$Gz = \frac{v \,.\, nn_1}{V} - HG \sin \varphi$$

$$= \frac{v \,.\, nn_1}{V} - HG \sin 10^\circ.$$

Setting out the Fig. 35 to scale, we obtain—

$$nn_1 = 13{\cdot}38 \text{ ft.}$$

$$v = \frac{10^2 \tan 10^\circ}{2} \times 300 = 2{,}644{\cdot}5 \text{ cubic ft.}$$

$$V = 300 \times 20 \times 5 = 30{,}000 \text{ cubic ft.}$$

$$\therefore \text{ G}z = \frac{2{,}644{\cdot}5 \times 13{\cdot}38}{30{,}000} - 2{\cdot}5 \sin 10^\circ = {\cdot}745 \text{ ft.}$$

Since GM sin 10° = 4·16 × ·1736 = ·724 ft., it will be seen that, had we assumed the upward pressure of the fluid through the centre of buoyancy at the inclination of 10° to pass through the metacentre, there would have resulted an error of only 3 per cent. in the value of righting arm.

Again at 20° of inclination we get

$$\text{G}z = 1{\cdot}58 \text{ ft.}$$

By continuing the calculation * at intervals of 20° we obtain :—

At 40°	..	..	..Gz = 2·81 ft.
,, 60°	..	..	.. ,, = 2·03 ,,
,, 80°	..	..	.. ,, = ·724 ,,
And at 90°	..	..	.. ,, = zero.

From the values of Gz obtained above we may draw a curve of "righting arms" to abscissæ values of angle of heel. The curve relating to this condition is shown in the diagram, Fig. 36, curve 1.

It will be noted (Fig. 35) that at the angle of heel of 26° 34′ the edge *d* (or *a*) is awash. For angles of inclination greater than this the rate of increase of righting arm and righting moment diminishes until an angle of inclination of about 35° is reached, at which we obtain the maximum value of righting arm, and thereafter the curve 1, Fig. 36, falls away easily to zero at 90° of inclination. The angle of inclination at which the righting arm becomes zero is designated the "Range of Stability."

If the inclination be continued beyond 90° the righting arm is of negative value, and this will continue until 180° of inclination is reached, at which the righting arm is again zero.

* After the edge *d* (or *a*) becomes immersed, the actual position of the centre of buoyancy may be readily determined geometrically by some such method as that referred to on p. 62 and illustrated by Fig. 37

This is shown by the broken curve, Fig. 36—drawn below the line of abscissæ—which is a duplicate of the unbroken curve 1

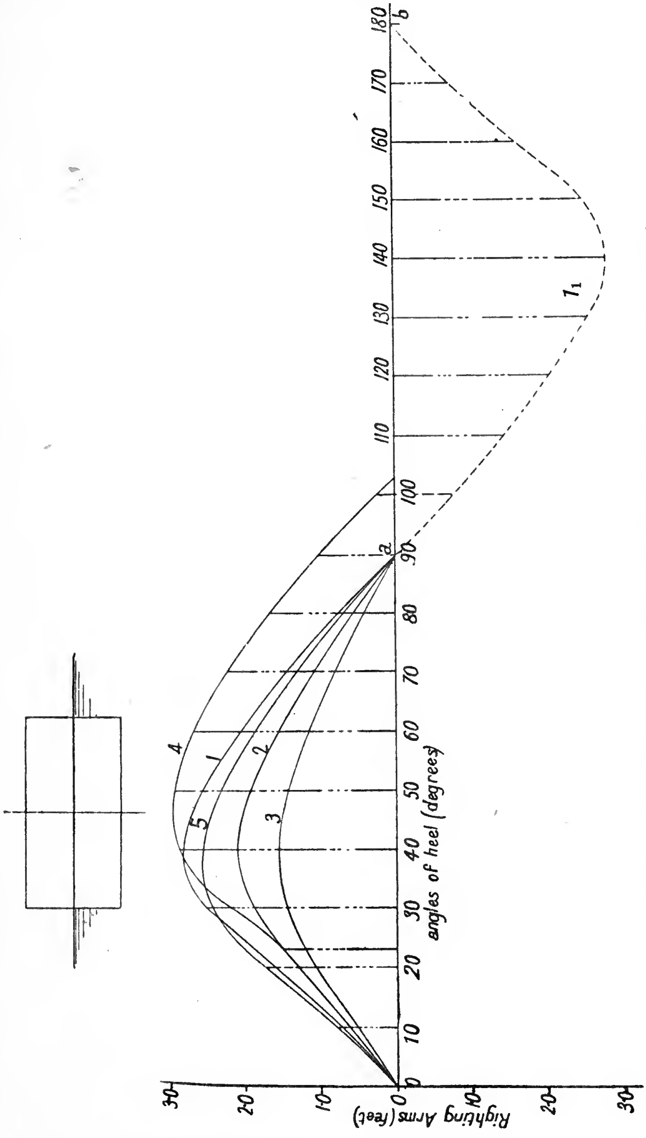


FIG. 36.—Statical Stability Curves for a Rectangular Prismatic Vessel.
Dimensions 300′ × 20′ × 10′.
C.G. assumed at 5·0′ above base.
Curves 1, 2, 3, refer to 5, 6 and 7 ft. draught respectively.
Curve 4 refers to 6 ft. draught but freeboard increased 2 ft.
" 5 " " " " beam " "

reversed, so that, for example, the ordinates at 30° and 150° of inclination are equal in value but opposite in sign. By

continuing the rotation of the vessel through 360°, so that the normal or initial position is again reached, the whole curve $ola l_1 b$ would be repeated.

It will be seen that the points o, a, and b indicate positions of stable, unstable, and stable equilibrium of the vessel respectively, and so on for the complete rotation, the positions of stable and unstable equilibrium following each other successively, confirming what was stated on p. 19 as being a "general law."

It is worth noting here that in the case of vessels of known geometric form, as with the present rectangular vessel, it is often much easier to obtain the centre of buoyancy, and therefore the righting arm, direct by setting out the section to scale than to employ Attwood's formula. For example, Fig. 37 represents the vessel inclined at an angle of 40°; it is required to obtain the position of the centre of buoyancy, *i.e.*, the centre of area of the trapezium WcdL.

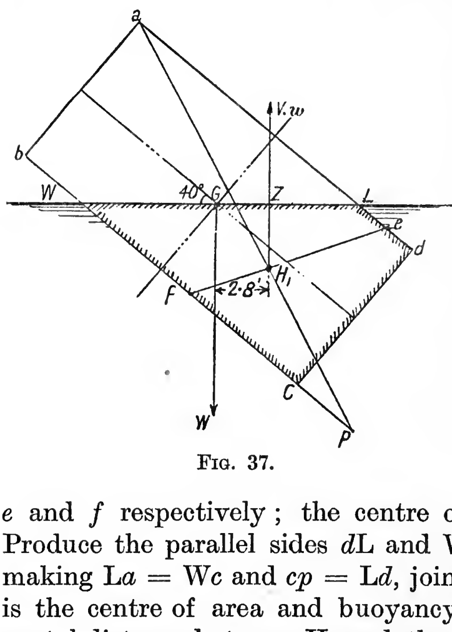


FIG. 37.

Bisect Ld and Wc in e and f respectively; the centre of area is situated in ef. Produce the parallel sides dL and Wc in opposite directions, making La = Wc and cp = Ld, join ap, cutting fe in H_1. H_1 is the centre of area and buoyancy required, and the horizontal distance between H_1 and the vertical through G is the righting arm, 2·8 ft., as calculated above.

Before leaving the rectangular-shaped vessel it will be instructive to investigate the effect of change of draught, or, by modifying the transverse dimensions, the effect of alteration of depth (and therefore of freeboard) and of beam, on the nature of the stability curve.

Curves 2 and 3, Fig. 36, are the stability curves referred to draughts of 6·0 and 7·0 ft. respectively, the position of the centre of gravity remaining unaltered.

The effect produced in reducing the value of "righting arm" and "righting moment" by augmenting the draught, or conversely by decreasing the freeboard, will be noted. An examination of the curves for any particular inclination, say 40°, will show the relative change in righting arm and righting moment; thus :—

Draught.	Displacement.	Righting Arm.	Righting Moment.
5·0 ft.	W.	2·8 ft.	2·8 W.
6·0 ft.	1·2 W.	2·14 ft.	2·57 W.
7·0 ft.	1·4 W.	1·55 ft.	2·17 W.

This might, in passing, be stated generally as follows :—For any particular angle of heel let "a" be the righting arm when the displacement is W, and let "$a \pm x$" be the righting arm when the displacement is $W \pm w$.

The ratio of "righting arms" is $\frac{a \pm x}{a} = k$.

The ratio of "righting moments" is $\frac{(W \pm w)\,(a \pm x)}{W \,.\, a} = k\left(1 \pm \frac{w}{W}\right)$.

Consider now the vessel floating at a draught of 6·0 ft. and (centre of gravity unaltered) to have the freeboard increased by 2·0 ft., so that the depth of vessel is 12·0 ft. It is evident that it will be necessary to incline her to a much greater angle (about 31°) before the edge "d" becomes awash than was the case (about 22°) with the normal vessel, with the result that as she heels the centre of buoyancy will move farther out or away from the centre of gravity before it begins to recede.

For inclinations beyond 22° the value of Gz will be greater than for the normal vessel, but will be identical for the two cases up to that angle. Moreover, at 90° of inclination there will be a positive value of righting arm, and the range of stability will be increased so that the curve 4 intersects the line of abscissæ at a point representing an inclination of about 104°.

Consider now the beam to be increased by 2 ft., the depth

remaining equal to 10 ft.; the result is indicated by curve 5 on the same diagram.

Comparing the three cases we find that with increased beam (curve 5) the righting arms are greater than in the case of the normal vessel (curve 2), and up to an angle of about 35° are greater, and for greater inclinations are less, than with increased freeboard (curve 4).

It may be stated generally that freeboard has a much greater effect in lengthening out curves of stability and augmenting the righting arm than has beam.

It should be remembered, however, that freeboard, beam, and depth are relative quantities, and must be considered with relation to draught of water; that the vertical position of centre of gravity and centre of buoyancy must be taken in comparison with the depth of vessel—all important factors separately and collectively influencing the design of the vessel.

With reference to this point it may be stated that, had the position of the centre of gravity been altered with a change in depth or beam, or with a modification in draught, instead of assuming a fixed position above base, the curves shown in Fig. 36 would undergo modification.

In practice a change in depth would almost invariably influence appreciably the vertical position of the centre of gravity, an increase in depth causing an increase in height of centre of gravity above base, and *vice versâ*. A change in beam, though having its effect on the position of the centre of gravity, would not affect the latter to the same extent as a change in depth. Again, a change in draught would be brought about by a change in the amount of cargo carried either in respect of bulk or specific gravity or both; in each case the position of the centre of gravity would be affected.

In any case, if $\pm x$ denote the change in vertical position of the centre of gravity above base, the change in value of "righting arm" for any particular draught and angle of heel would be

$$\pm x \sin \varphi,$$

where φ denotes the angle of heel.

Passing on to vessels of ship-shape form, we shall find generally that the problem of stability is more complex than

in the case of the rectangular vessel just considered, or of other bodies of regular geometric form. To attempt the solution by direct calculation or geometrically would involve an amount of labour and time which could be ill spared in the designing office ; consequently mechanical means are generally resorted to, which, with care, give results very closely approximating to the truth.

Several methods have been proposed by which to determine mechanically the stability of a vessel, and have all found their origin in the desire to obviate as far as possible the labour which would be involved in the case of direct calculation or geometrical computation.

Amongst these, Heck's mechanical tank and later methods * rank high, as being both accurate in principle and easy to understand by the ordinary draughtsman.

Amsler's Integrator is now almost universally employed where stability calculations have to be made. Applied to the sections or body plan of a ship, this instrument records both the areas and the moments of those areas about a certain axis —to be decided on in each case—at one operation, from which the levers or perpendicular distances of the centres of area from that axis may be determined, and in combination with Simpson's Rules the levers for the vessel may be estimated. If the axis be so placed as to pass through the point representing the actual centre of gravity of the vessel, it is clear that the lever obtained will give the righting arm in the stability calculation.

Generally, however, it would not be convenient to take the axis to pass through the centre of gravity, even if the position of the latter were known at that stage of the design at which the calculation is made ; consequently a correction has to be made after the position of the centre of gravity has been determined, either by calculation from the finished ship, or by an inclining experiment on completion of the vessel.

The calculation by means of the Integrator will be explained later when we come to consider the question of Cross Curves of Stability.

A ship under the action of external forces may be inclined in

* Trans. I. N. A., 1885–6–7 ; and Proc. N. E. C. Inst. Engineers and Shipbuilders, 1903–4.

any direction; the inclination may be either directly transverse or longitudinal, or in any intermediate or skew direction.

It may be assumed that if the stability in the transverse and longitudinal directions be satisfactory, then for any intermediate direction it is also satisfactory; consequently it is usual to determine only the transverse and longitudinal stability. Of these it will be found that the latter is very much in excess of the former, and therefore it is of the greater importance to determine precisely the "transverse stability of ships."

With reference to initial stability (GM), the difference in value for the transverse and longitudinal directions may be seen from the formula for height of metacentre above centre of buoyancy, viz.,

$$\text{HM} = \frac{\text{Moment of inertia water plane area}}{\text{Volume of displacement}},$$

given on p. 25.

Now, clearly the moment of inertia of water plane area for an inclination in the longitudinal direction is usually very much in excess of that in the transverse direction, hence the value of HM, and therefore of GM, is correspondingly greater in the one direction than in the other, the volume of displacement remaining the same.

It will be found later that longitudinal initial stability is of great importance, inasmuch as it has a direct bearing or influence on the *trim* of the ship, and as affecting her pitching motion at sea.

For the present, however, we will confine our remarks to the more important case of stability in the transverse direction. From the lines of the vessel we are able to determine the displacement and position of centre of buoyancy by the usual method of computation by the aid of one or more of Simpson's Rules, and the moment of inertia of water plane area may be calculated in a similar manner. In this way the value of HM and the height of metacentre above the baseline or underside of keel is determined from which to construct the metacentric diagram described on pp. 55—56.

For inclinations not exceeding about 8° from the normal position of equilibrium the metacentre may be regarded as practically a fixed point, and within this range the righting

arm Gz is equal to GM . θ, where θ denotes the circular measure of the angle of heel.

The angle to which most ships would incline in *still water* under the action of a steady external force, as, for example, the action of wind on the sails or upper works, would lie within this range ; consequently the " stiffness " of a ship or " her power to resist inclination from the normal by the action of a steady external force " may be measured by GM . θ, her moment of resistance being W . GM . θ, where W is the weight of the vessel and θ lies within the limit to which the metacentric method applies. If the value of GM be small so that the vessel may be easily inclined under the action of external force, she is termed " crank " or " tender."

On the other hand, a " steady " ship is one which does not depart far from the upright in *disturbed water*, *i.e.*, under the action of waves at sea.

It often occurs that *stiff* ships are the *least steady* and that *crank* ships are the most *steady* at sea.

The question of steadiness really belongs to the department of " rolling of ships " ; reference may be made, however, to the formula given on p. 44, relating to the time of roll. If the time of roll T be great compared with the period of the wave, the tendency is for the vessel to retain the upright position, for the reason that if she be momentarily situated on the slope of a wave, so that she tends to incline from the upright, she will almost immediately be oppositely affected by the reverse slope of the approaching wave tending to restore her to the initial position. This effect naturally will be greater, the greater the ratio of the period of the ship to that of the wave, which means that the " metacentric height " must be small. The reverse effect would be produced in a vessel of comparatively large value of GM.

The danger of stiffness, apart from the consequent discomfort to passengers and the living freight, is that by its severe or jerky motion the structure might be strained and fittings loosened ; this naturally would be most marked at those parts of the structure the farthest removed from the axis about which the vessel is rolling, due to the greater velocity acquired, and the change of motion at the end of each roll.

It is evident that the relative positions of the centre of gravity

and metacentre is a matter of great importance, and the assignment of a suitable value of GM therefore concerns the naval architect very intimately; it is a matter in which he has to exercise a wise discretion and has to call upon the best experience at his command. But while it is the object at least to secure a positive metacentric height for the upright condition, it does not follow that its absence would render the vessel dangerous or unsafe, although such a condition might cause momentary alarm, and should be guarded against when the vessel is prepared for sea. The presence of a small *negative* value of GM would cause the vessel to incline from the upright position, but generally the inclination would not proceed far before a position of stable equilibrium would be reached, beyond which the righting arm would be of such magnitude as to render the possibility of upsetting out of the question. Indeed, in many cases ships, especially those of the *tramp* class, are so situated in the *light* condition, and will only rest in a position of stable equilibrium after inclining to a small angle on either side of the upright. Where such vessels are required to proceed to sea in the light condition they do so in ballast, which, generally being situated low down, affects the value of GM advantageously.

Metacentric height cannot be regarded as indicative of the stability qualities of the vessel unless conjointly with it we take into consideration the type and form of the vessel to which it refers, for whereas the value of GM is governed by the form of the under-water portion of the ship for the upright condition, statical stability—that quality which must be appealed to at considerable angles of heel—is governed by both the under-water and the exposed or above-water parts of the structure.

We saw when dealing with the rectangular or box-shaped vessel how the character of the stability curve (Fig. 36) was at once changed by increasing the freeboard; how, generally, the righting arms were increased and the range of stability enhanced, notwithstanding that the metacentric curve remained *unaltered*—provided that the position of the centre of gravity is not raised. On the other hand, by increasing the beam it was shown that the increase in righting arm was accompanied by an increase in initial stability, the range of stability remain-

ing unaltered. It is clear, then, that in two vessels of different dimensions, although the initial stability might in each case be the same, yet the value of the righting arm at any particular angle of inclination, and the range of stability, might be widely different; also, that if by altering the dimensions of a vessel the initial stability were increased, it does not follow necessarily that at large angles of inclination there would result an increase of righting arm and range of stability.

What is true of the vessel of regular geometric form is also true in principle of the body of ship-shape form; but in the latter case the problem is much more difficult, due to the non-uniformity of the outward shape of the body which we are compelled to adopt in order to satisfy all the conditions of speed, strength, service, etc., to be fulfilled in the finished design.

Initial stability, therefore, is no criterion of the other and more important characteristics of righting arm and range. One type of vessel might be quite as safe with 9 in. of GM, and probably more comfortable, as another with 2 or 3 ft. The amount to be assigned in each case will depend largely upon the service for which the vessel is intended, and the form and dimensions of the vessel will in a great measure be determined by the nature of that service.

For example, vessels of low freeboard and substantial superstructure would generally be assigned an amount of initial stability much greater than with vessels of normal proportions. The former include warships having heavy armour and armaments carried at and above the water line, and shallow draught steamers, such as are used for river and lake service, with heavy superstructure in the shape of decks with passenger accommodation; while ordinary seagoing cargo and passenger vessels with fairly deep draught and weights equitably distributed form examples of the latter class. In the one case the metacentre and centre of gravity are situated comparatively high, while in the other they are fairly low.

Obviously, where the centre of gravity G is situated high up, the horizontal distance between G and H (centre of buoyancy) will, as the vessel inclines, diminish at a much greater rate than in the case in which G occupies a low position, and will become zero at a smaller angle of inclination; indeed, it may be taken

as generally true that, assuming the same distance between G and M, the value of the righting arm will diminish by raising the positions of these points and increase by lowering them, and the range of stability will be similarly affected. It is for this reason that where a particular type of vessel is such that a high position of centre of gravity is inevitable it is necessary to so execute the design as to bring the metacentre well above the centre of gravity to an extent much greater than would be the case were G situated low down, so that a sufficient value of righting arm might be preserved as the vessel inclines from the

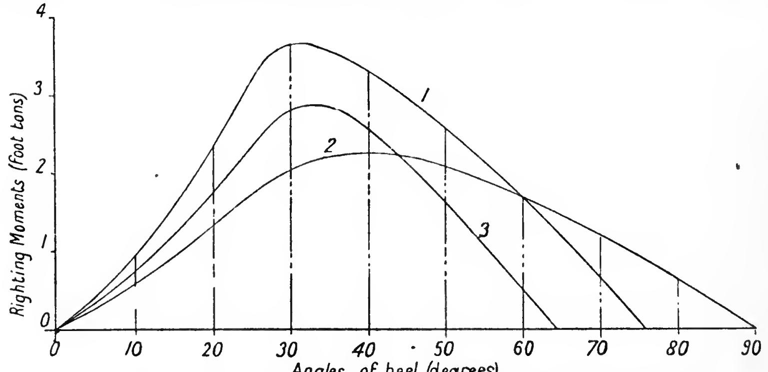


Fig. 38.—Comparative Statical Stability Curves for Rectangular Vessels.
Curve 1.—Dimensions 300′ × 26′ × 12′. Draught 6·0 ft. C.G. above base 8·0 ft.
" 2.— " 300′ × 20′ × 10′. " " " " 5·0 "
" 3.— " 300′ × 26′ × 12′. " " " " 9·0 "
Weight of vessel 300′ × 20′ × 10′ = W.
Scale for moments — one division = W foot tons.

upright. This may be well illustrated by again considering the rectangular or box-shaped vessel.

Suppose the height of centre of gravity above base to be increased from 5 to 8 ft. while the dimensions of the vessel are altered from 300 × 20 × 10 ft. to 300 × 26 × 12 ft.

Let the draught in each case be 6·0 ft., so that the displacement of the vessel is increased from W to 1·3 W, and the initial stability, or GM, from 3·55 to 4·39 ft.

Fig. 38 shows the stability curves for these two cases, curve 1 representing the righting moments for the vessel of new dimensions, whereas curve 2 refers to the original or normal vessel. It is seen that, notwithstanding the increase in freeboard and beam of 2 and 6 ft. respectively and the augmenta-

tion of the initial stability by ·84 ft., the statical stability vanishes at an angle of 76°, and beyond about 40° of inclination there is a rapid falling-off in the value of righting moment as compared with the normal vessel. Had the centre of gravity been raised to 9 ft. instead of 8 ft., so that the value of the initial stability is now 3·39 ft.—*i.e.*, practically identical to the case of the normal vessel—the righting moments, curve 3, would have been somewhat greater up to about 42° of inclination, after passing which there would have been a rapid falling off to zero at an angle of 63°.

In the foregoing simple cases of the rectangular vessel we have considered separately the effect of form or dimensions on stability, (*a*) assuming a constant height of centre of gravity above base, Fig. 36, and (*b*) on the assumption of certain variations in the vertical position of centre of gravity, Fig. 38. With actual seagoing vessels, however, the position of the centre of gravity is not only variable, but its path relative to a fixed baseline is, generally speaking, very irregular. Unlike the curve of metacentres, no curve representing the variation of vertical height of centre of gravity with draught can be drawn. The position varies with every change in magnitude and position of the weights which contribute to the equipment and lading of the vessel, and as some of these items, as, for example, stores and fuel, undergo a continuous change when the vessel is at sea, they must exercise a corresponding influence on the centre of gravity. What is done in practice is to determine the position of G for certain definite assumed extreme conditions of loading and satisfy ourselves that the stability under such conditions is satisfactory; then presumably for other possible conditions the stability may be regarded as satisfactory. If, however, at any time a serious departure from these conditions of loading be contemplated, a special calculation of the vessel's stability would require to be made.

The greatest trouble is experienced with vessels primarily designed for the carrying trades, where, in addition to the effect produced by such variable items as fuel and stores, there is the more potent factor of the variability in nature and amount of the cargo, with which the vessel may be loaded from port to port. In those cases of ships engaged in specific trades where the nature of the freight is hardly, if ever, varied

the difficulty is reduced to a minimum, but with vessels employed in carrying general cargoes it is considerably increased.

In the case of warships and yachts the problem is very much simplified, inasmuch as after the design is completed and the centre of gravity of the finished ship has been definitely assigned the only way in which the latter can be affected is by the consumption, or increasing the quantity, of fuel and such variable items as water and stores.

In all cases the centre of gravity of the vessel in the *light* condition—*i.e.*, fully equipped, with no coal in bunkers, with or without water in the boilers, and holds entirely empty—should be carefully estimated, from which may be derived the position of centre of gravity, and the resultant stability for all seagoing conditions of the vessel.

To illustrate this point we will consider a further example of a box-shaped vessel of dimensions

$$200 \times 18 \times 12 \text{ ft.}$$

floating when light at a draught of 4 ft. in sea water, the centre of gravity being situated 6 ft. above base. The vessel is to be loaded to a draught of 8 ft. with 150 tons of coal in bunkers, the centre of gravity of which is estimated at 7 ft. above base, the remainder of the freight consisting of cargo and stores, the centre of gravity of which is not to exceed 5 ft. above base.

Calculate the effect on the centre of gravity and determine the initial stability in the light, and the minimum, initial stability in the loaded condition.

(*a*) Weight of light ship $= \dfrac{200 \times 18 \times 4}{35} = 411{\cdot}5$ tons.

Height of centre of buoyancy of light ship above base $= 2{\cdot}0$ ft.

Distance between centre of buoyancy and

$$\text{metacentre} = \frac{\frac{1}{12} \cdot 200 \cdot 18^3}{200 \times 18 \times 4} = 6{\cdot}75 \text{ ft.}$$

Height of metacentre above base	8·75 ,,
Centre of gravity above base	6·0 ,,
Initial stability of light ship =	2·75 ,,

(*b*) Weight of loaded ship $= \dfrac{200 \times 18 \times 8}{35} = 823$ tons.

,,	,,	light ship	= 411·5 ,,
,,	,,	cargo, stores, etc.	= 411·5 ,,
,,	,,	coal	= 150·0 ,,
,,	,,	cargo and stores	= 261·5 ,,

To determine the centre of gravity of the loaded ship :—

				foot tons.
Weight of light ship	411·5 tons,	C.G. above base 6·0 ft.,	moment	= 2469·0
,, coal	150·0 ,,	,, ,, 7·0 ,,	,,	= 1050·0
,, cargo and stores	261·5 ,,	,, ,, 5·0 ,,	,,	= 1307·5
,, loaded ship	823 ,,			823)4826·5
				5·86

or *maximum* height of C.G. of loaded ship above base = 5·86 ft.

Now :—Height of centre of buoyancy above base = 4 ft.

Height of metacentre above C. B. =

$$\frac{\frac{1}{12} \cdot 200 \cdot 18^3}{200 \cdot 18 \cdot 8} = 3{\cdot}37 \text{ ,,}$$

Height of metacentre above base . .	7·37 ,,
Maximum height of C. G. ,, ,, . .	5·86 ,,
Minimum initial stability loaded ship . .	1·51 ,,

The centre of gravity for the light condition—if not actually calculated from the lines of the vessel, the plans showing the scantlings of material, and the general arrangement to which the vessel is to be built—may be approximately estimated from the known position in the case of other similar vessels already built ; this often suffices for the purpose of the preliminary design, but when the design is completed a more exact calculation should be made, and after the vessel is built and nearly or quite completed for service should be checked by subjecting the vessel to an "Inclining Experiment."

This experiment is carried out in the following manner :—

The vessel is prepared by first removing all superfluous weights, such as workmen's materials, tools, etc., which do not form part of the equipment. The bunkers should be entirely empty, or, if they contain coal, a careful estimate of

the weight and the centre of gravity of its mass must be made after it has been trimmed, so as to prevent any movement with that of the ship as she inclines. All water tanks, whether for ballast or otherwise, should be completely full or empty, and the boilers preferably empty. All fittings, such as hawser-reels, boats, spare gear, etc., should be placed in their correct

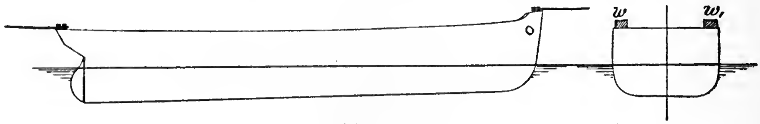


FIG. 39.

position or otherwise carefully noted and allowed for in the calculation. Indeed, all possible care must be taken so that the result of the calculation will give an accurate idea of the vessel in her finished condition.

Certain weights—usually a number of standard 56 lb. weights—are placed on board on the upper or bridge deck amidships, or at other convenient positions, the same number on each side of, and generally in corresponding positions from, the middle line, as indicated by w and w_1, Fig. 39.

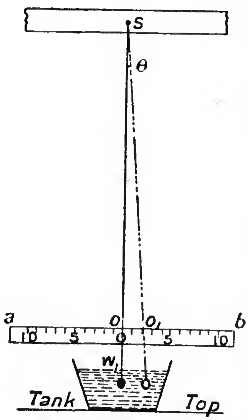


FIG. 40.

Two plumb-bobs * are suspended at chosen positions, one forward and the other abaft amidships, usually at about one-fourth the length of the vessel from the ends, the points of suspension being situated fairly high up, say at the main deck, and the bobs are so hung that they can swing freely without interruption across a horizontal batten ab, Fig. 40, fixed low down, and either graduated or so arranged that measurements can be made from the point o, where so denotes the initial position of the plumb-line with the vessel in the upright position.

The vessel is now taken into open water, clear of wharves or any obstacle which might cause an impediment to its

* Theoretically one plumb-bob would be sufficient, but for the purpose of checking the readings, more than one are necessary.

movement, and is moored *freely* fore and aft in the direction of the wind (if there be any), so that the action of the latter to produce or tend to produce transverse inclination may be reduced to a minimum.

The men on board required for manipulating the weights and generally to assist in carrying out the experiment take up certain specified and fixed positions. The draughts forward and aft are carefully noted and from the mean draught —or if the vessel trims, by special calculation—the displacement is obtained from the displacement diagram already constructed.

The vessel is now inclined by moving one set of the ballast weights, say w, from one side to an exactly defined or corresponding position on the other side of the deck, the horizontal distance oo_1, through which the plumb-line has moved over the batten ab, is noted, and the angle of inclination determined from the equation

$$\tan \theta = \frac{oo_1}{so}.$$

The weights first shifted are then replaced, when the plumb-line should take up its original position so; if it be found not to do so the deviation is noted.

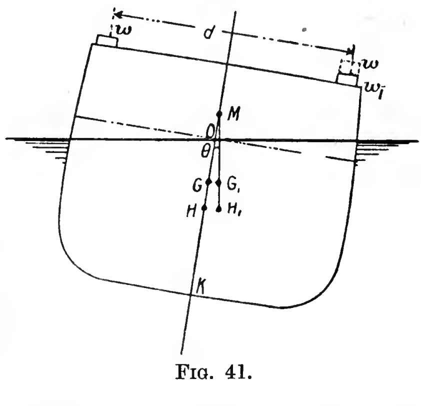


Fig. 41.

A similar operation and observation is gone through with regard to the other set of ballast weights w_1.

We thus get four readings of the batten ab from which to determine the *mean* angle of inclination produced by the movement of the weight w or w_1 across the deck through a known distance d (Fig. 41).

Sometimes only one half of the weights w and w_1 are moved at a time and readings taken at each movement, in which case we would have eight readings to observe instead of four; the bob usually swings in a bucket of water in order to steady its movement (w_1, Fig. 40).

The result of the experiment may now be used to determine the position of the centre of gravity of the vessel

for the condition in which she is inclined as follows (see Fig. 41) :—

The movement of the centre of gravity G perpendicular to HG (*i.e.*, parallel to ww_1) is given by

$$GG_1 = \frac{w}{W} \cdot d = GM \tan \theta,$$

$$i.e., \ GM = \frac{w \cdot d}{W \tan \theta}.$$

where W is the displacement of the vessel and M the metacentre, θ being small.

Now the position of the centre of buoyancy H is known or may be calculated from the lines of the vessel if there is considerable trim, and :—

$$HM = \frac{\text{Transverse moment of inertia of water plane area}}{\text{Volume of displacement}}.$$

$\therefore$ KH + HM = KM may be determined or read off from the metacentric diagram. Hence KM — GM = KG is known, giving the height of the centre of gravity G above base or bottom of keel.

By carrying out the experiment carefully the position of G may be estimated to a very close degree of approximation, and, after correcting for the effect of w, w_1 and men, etc., on board, the position of the centre of gravity of the vessel for the light condition may be determined.

In illustration is given below details of the calculation relating to an inclining experiment made by the author on a torpedo-boat destroyer.

Report :—

Condition of weather—dull. Light wind on port bow.

Condition of vessel when inclined :—Bunkers, boilers, and fresh water tanks empty. Bilges dry. Reserve feed tank full. Feed tank empty. Distiller and condenser dry. Vessel practically complete.

Men on board 38 in number, placed at various positions.

Inclining weights, 4 tons—2 tons each side on deck, placed 10 ft. apart centre to centre in two batches.

Displacement as deduced from mean draught = 259·5 tons.

The Experiment.

		ft. in.		ft. in.
Length of plumbs	Forward	14 6	After	14 $2\frac{1}{4}$
Reading 1st shift—2 tons—upright to starboard	,,	$4\frac{3}{4}$	,,	$4\frac{5}{8}$
,, 2nd ,, — ,, —starboard to upright	,,	$4\frac{5}{8}$	,,	$4\frac{1}{4}$
,, 3rd ,, — ,, —upright to port	,,	$4\frac{3}{8}$	,,	$4\frac{3}{8}$
,, 4th ,, — ,, —port to upright	,,	$4\frac{3}{8}$	,,	$4\frac{3}{8}$
Mean readings	,,	·377 ft.	,,	·367 ft.

$$\left.\begin{aligned}\text{Mean value of GM by forward plumb} &= \frac{2 \times 10 \times 14{\cdot}5}{259{\cdot}5 \times {\cdot}377} = 2{\cdot}96 \text{ ft.}\\ \text{,, ,, ,, after ,,} &= \frac{2 \times 10 \times 14{\cdot}187}{259{\cdot}5 \times {\cdot}367} = 2{\cdot}98 \text{ ft.}\end{aligned}\right\} \begin{array}{l}\text{mean}\\ 2{\cdot}97 \text{ ft.}\end{array}$$

Height of metacentre above base (from diagram)	=	10·31 ft.
Mean value of GM (inclining experiment)	=	2·97 ,,
Height of centre of gravity above base	=	7·34 ,,

Calculation.

of "centre of gravity" of vessel complete with all weights on board and 40 tons of coal in bunkers.

	Tons.	Vertical. Levers above base.	Vertical. Moments.
Deductions :—			
Inclining weights	4·0	13·60	54·400
38 men	2·54	10·63	27·000
Water in feed tank (when vessel inclined)	3·90	1·75	6·825
Gear, etc.	·65	12·35	8·027
Total deductions	11·09		96·252
Additions :—			
Water in boilers, feed tank, distillers, evaporators, condensers, and circulating pumps and pipes.	11·15	5·35	59·653
Fresh water in ship's tanks	3·144	9·80	30·811
Engineers' stores	1·38	3·47	4·788
Naval stores	4·50	3·80	17·100
Deck gear, fittings, etc.	3·03	13·20	39·996
Boats	1·50	16·93	25·395
Crew, effects and provisions	12·44	8·60	106·984
Arms and armaments	13·42	13·42	180·096
Coal in bunkers	40·00	5·50	220·000
Total additions	90·564		684·823
Total deductions	11·09		96·252
Nett additions	79·474		588·571
Vessel as inclined	259·5	7·34	1904·730
Vessel complete with 40 tons of coal	338·974	7·35	)2493·301

Mean draught for this displacement	=	5 ft. $9\frac{1}{4}$ in.
Height of metacentre above base for this displacement	=	9·83 ft.
Height of centre of gravity for this displacement	=	7·35 ,,
Metacentric height	=	2·48 ,,

A second experiment carried out on this vessel some six months after the first, when the vessel was completed and clear of all superfluous material, confirmed the above figures.

The actual result obtained was :—

Vessel complete with 40 tons of coal = 339·85 tons.
Metacentric height = 2·44 ft.

If at the time of making an inclining experiment the trim of the vessel differs to any considerable extent from that to which she is designed, so that the water plane is not parallel to that for the designed condition, the position of the metacentre above centre of buoyancy, and that of the centre of buoyancy relative to the baseline or underside of keel, must be specially calculated from the lines of the vessel, and not read off from the Displacement and Metacentric diagrams, which have been constructed from calculations made to the designed trim.

That this is so must be at once obvious when we consider that not only the submerged body but the area and moment of inertia of the water plane are different to what would have been the case had the vessel floated at the designed trim.

It has been pointed out that the combination of the curve of metacentres with the estimated position of the centre of gravity under different and specific conditions of loading is of itself insufficient as a criterion of the stability of the vessel at all possible angles of heel which she might assume, and, therefore, we are compelled to make independent and more exhaustive inquiry into the stability for angles of inclination beyond those to which the metacentric method applies.

For any one condition of draught, displacement, and inclination Attwood's formula, given on p. 27, might be used to obtain the righting arm or righting moment. The process, however, is laborious : it resolves itself into determining the correct position of the new water plane at the given inclination, so that the two wedges—of immersion and emersion—are of equal volume ; this preliminary part of the calculation is essentially tentative, and the exact position of the new water plane can only be quickly arrived at from a close experience of the form of ships in the designing office. When it is obtained the volume of either wedge and the distance between the centres of gravity of each have to be determined by the application of Simpson's or other rules, and used in Attwood's formula for the determination of the righting arm.

This process would have to be gone through for several angles of inclination at intervals of some 10° to 15° until the

stability vanishes, and from the results so obtained a curve of stability showing the variation in the righting arm or righting moment with angle of heel, constructed.

For each individual and distinct condition of loading a similar calculation would have to be made, and as there are generally at least three or four conditions to be worked out, ranging from the light to the maximum or load draught, at which particulars of the vessel's stability are required, it is obvious that the whole work of calculation would be prolonged and tedious, ranging over two or three weeks in the designing office. Even then the information, though complete in itself

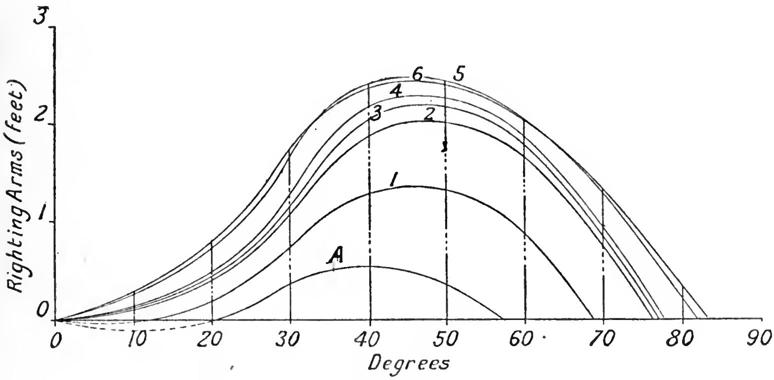


Fig. 42.

Statical Stability Curves for a Twin-screw 1st Class Passenger Steamer.

			Displacement. tons.	C of G above base. ft.
Curve 1.	Vessel light with water in boilers		9150	25·8
,, 2.	,, in docking condition (half stores, fresh water and 400 tons coal)		9650	24·74
,, 3.	,, at end of homeward voyage		10400	24·42
,, 4.	,, ,, ,, outward voyage		11200	24·13
,, 5.	,, ,, beginning of homeward voyage		13700	23·24
,, 6.	,, ,, ,, ,, outward voyage		14450	23·08

for each particular condition, is not sufficient to give all that may be required with regard to other conditions of loading without some modification, for the reason that the centre of gravity would invariably (except by coincidence) occupy a different position in each case, and consequently the righting

moments, being taken about a point which is variable in position, could not be compared without first correcting for a common centre of gravity.

This will be better understood by reference to Fig. 42, showing curves of statical stability relating to a modern first-class twin-screw passenger steamer in respect of conditions of displacement and positions of centre of gravity indicated thereon. Now, suppose we required to know the value of the righting arms at, say, 30° of inclination for a displacement and position of centre of gravity other than that to which any one of these curves corresponds, we would proceed as follows :—

Assume the new displacement to be 12,000 tons and that the centre of gravity of the vessel at this displacement has been estimated at 23·5 ft. above base.

From Fig. 42 we obtain the following particulars for the given angle of inclination, 30° :—

From Fig. 42.			Correction for Righting Arms reduced to 23·5 ft. C of G.	Corrected Righting Arms referred to 23·5 ft. C of G.
Displacements.	C of G above base.	Righting Arms.		
tons.	ft.	ft.	ft.	ft.
9150	25·8	·72	(25·8 —23·5) sin 30° = + 1·15	·72 + 1·15 = 1·87
9650	24·74	1·06	(24·74—23·5) sin 30° = + ·62	1·06 + ·62 = 1·68
10400	24·42	1·12	(24·42—23·5) sin 30° = + ·46	1·12 + ·46 = 1·58
11200	24·13	1·21	(24·13—23·5) sin 30° = + ·31	1·21 + ·31 = 1·52
13700	23·24	1·63	(23·24—23·5) sin 30° = — ·13	1·63 — ·13 = 1·5
14450	23·08	1·74	(23·08—23·5) sin 30° = — ·21	1·74 — ·21 = 1·53

From this table of corrected righting arms, construct a curve to ordinate values of righting arm, and abscissæ values of displacement, as shown in Fig. 43.

By the aid of this curve we may obtain the value of righting arm corresponding to any displacement and draught for the specified angle of inclination of 30°, the centre of gravity being assumed at 23·5 ft. above base throughout.

Thus for the particular displacement of 12,000 tons we find the righting arm corresponding thereto is 1·5 ft.

The curve shown in Fig. 43 is termed a "cross curve" of stability, and to be of service it would require to be used in

conjunction with other similar curves similarly constructed and corresponding to other angles of inclination, say at intervals of 10° or 15°. From such a series of curves we could construct a complete curve of statical stability for the vessel at any displacement and height of centre of gravity above base 23·5 ft.; further, by making a correction similar to that shown in the table, p. 80, for a change in position of the centre of gravity, statical stability curves may be constructed corresponding to any displacement and position of centre of gravity.

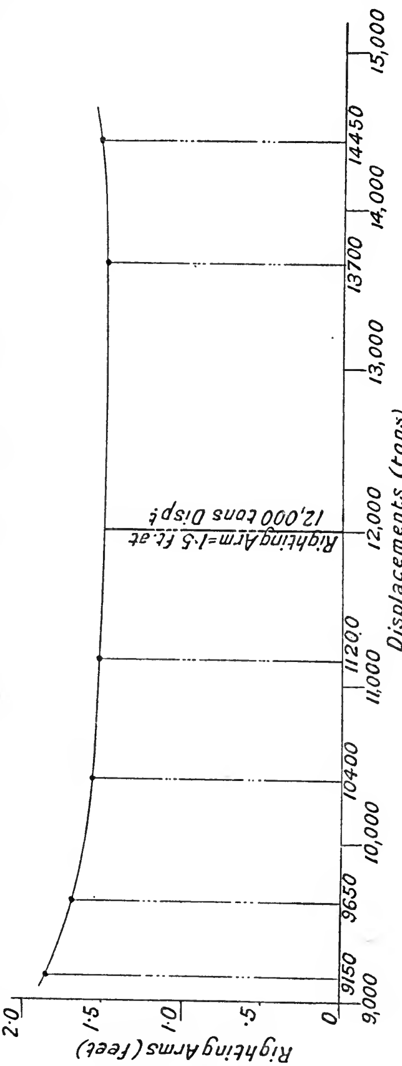


FIG. 43.—Curve of Righting Arms relative to Displacement of First-Class Twin-Screw Passenger Steamer referred to in Fig. 42, the Vessel being inclined at 30° and the C.G. assumed 23·5 ft. above base.

To adopt this method would not only be tedious, but would involve more time than generally could be spared; and it is found to be not only more simple but more expeditious to commerce by constructing "cross curves" by the aid of "Amsler's Integrator," thence deducing the particulars necessary for constructing the required curves of statical stability.

Such cross curves are generally got out before the vessel is completed, and therefore before its centre of gravity as finally determined by the inclining experiment is known. It is usual to assume the centre of

gravity as occupying a certain position above base and to remain fixed for the whole of the calculation; afterwards the necessary correction is made when the centre of gravity of the vessel for the particular condition in which she will be floating is known.

The method usually adopted is concisely outlined below. The body plan of the vessel is constructed to the complete

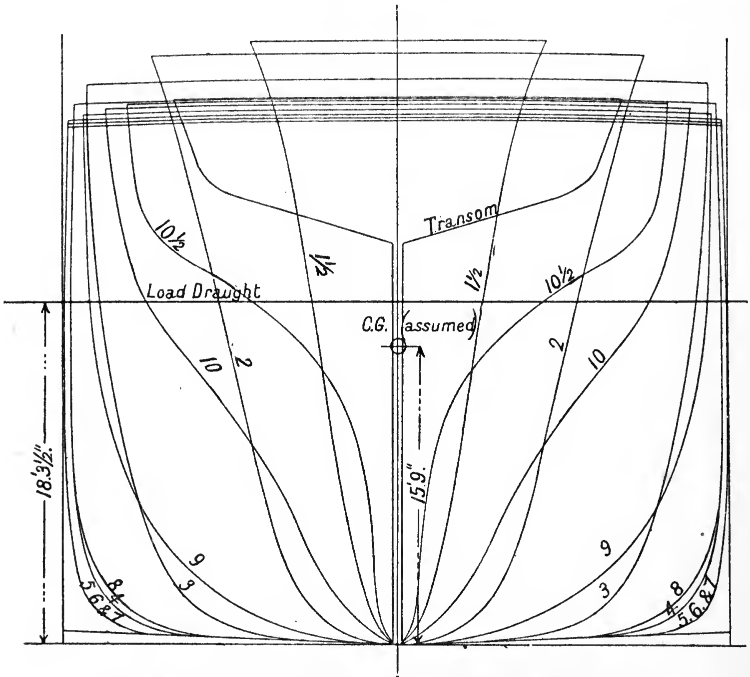


FIG. 44.—Displacement Sections for Stability Calculations.
Dimensions of vessel 270′ × 36′ ext. × 20′·5 M*d*.
N.B.—Sections taken to height of erections. Ht. of erections 7′·5.

or full *displacement* sections * as shown in Fig. 44, it being advantageous to draw the half, even, and odd sections in distinctive colours respectively, and the position of the assumed centre of gravity is marked on the plan.

A sheet of tracing-paper is taken and the middle line (referred to hereafter as the zero line) and parallel water lines

* Where the shell or deck plating consists of inside and outside strakes, the sections are drawn to 1·5 times the mean thickness of plating outside the frame and beam lines.

(five or six in number), to which the displacements are to be calculated, are traced and the position of the centre of gravity marked thereon. Straight lines, radiating at 10°, 20°, 30°, etc., from the middle line are drawn through the point corresponding to the centre of gravity, Fig. 45.

To determine the displacements and righting arms for, say, the inclination of 10° :—The tracing-paper, Fig. 45, is placed

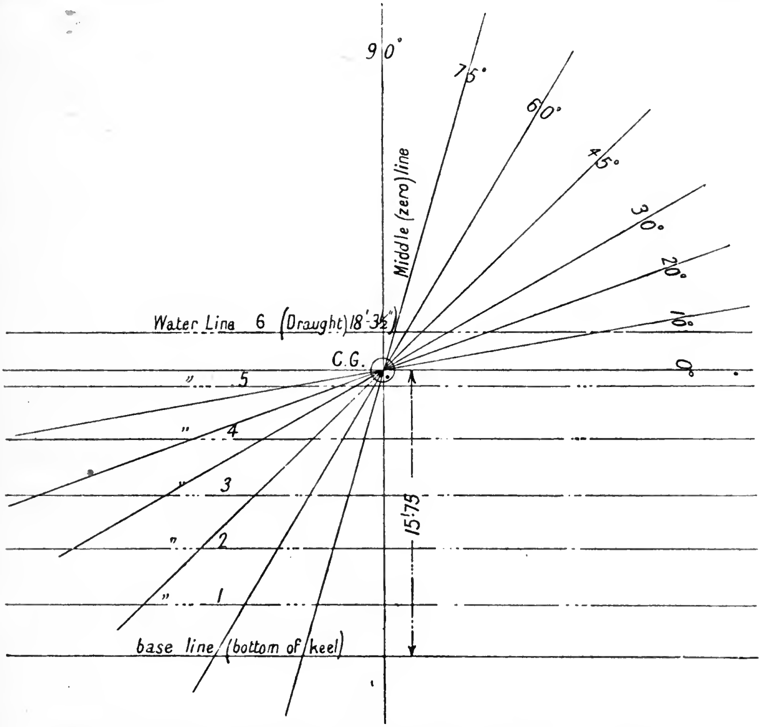


Fig. 45.—Lines of Inclination for use with Integrator and Fig. 44. Water lines spaced 3'·05 apart.

so that the point G (centre of gravity) coincides with the corresponding point on the body plan when laid upon it, and with the 10° line coinciding with the middle line of the body plan. This is done by rotating the latter while the tracing-paper remains fixed, so that for each angle of inclination the zero line represents the plane about which moments are to be taken.

The calculation may now be made by placing the bar of

the integrator parallel to the zero line, and at a fixed distance therefrom determined by the distance pieces (see Appendix). The half-sections are first traced over with the pointer of the integrator up to the particular water line under consideration, and the sum-readings of the areas and the moments of those areas about the zero line are read off from the counters or indicators of the instrument; next the even and finally the odd sections are similarly traced.

Let the area readings be denoted respectively by a, b, c.
,, moment ,, ,, ,, a_1, b_1, c_1.

Using Simpson's first rule* we proceed according to the following table :—

Area Readings.	Function of Areas.	Simpson's † Multipliers.	Moment Readings.	Functions of Moments.
a	a	1	a_1	a_1
b	$2b$	2	b_1	$2b_1$
c	$4c$	4	c_1	$4c_1$
Sum of functions	S			S_1

Then $\dfrac{S \times k \times d}{3 \times 35}$ = the displacement to the water line in tons,

and $\dfrac{S_1 \times k_1}{S \times k}$ = the righting arm or lever for the particular inclination (10°) and water line considered,

where d is the distance apart of the sections of the vessel and k and k_1 are the constants of the instrument to be used for converting the readings into actual areas and moments, partly

* For the method of computation by Tchebycheff's Rules, see Appendix to Chapter II.

† The multipliers 1, 2, 4 used here are not strictly correct—*e.g.*, taking *nine* ordinates equally spaced with two half-ordinates between the first and the last two the complete order of multipliers would be :—$\frac{1}{2}$, 2, $1\frac{1}{2}$, 4, 2, 4, 2, 4, $1\frac{1}{2}$, 2, $\frac{1}{2}$, of which that for the half-ordinates would be 2, for the even ordinates $1\frac{1}{2}$, 2, 2, $1\frac{1}{2}$, and for the odd ordinates $\frac{1}{2}$, 4, 4, 4, $\frac{1}{2}$. Now the end (first and last) ordinates are generally negligible, and no appreciable error would accrue by neglecting them and thus taking "4" as the multiplier for the odd ordinates throughout. Again, by taking the multipliers for the half-ordinates as "1" instead of "2" and for the even ordinates "2" throughout, it is found that no appreciable error results, and the multipliers as given above, while they considerably facilitate the calculation, give results which are practically accurate. To be quite accurate, however, four multipliers should be taken, viz. :—

$\frac{1}{2}$ for the *end* (first and last) ordinates.
$1\frac{1}{2}$,, *second* ordinates from the ends.
2 ,, *half* ordinates together with the even ordinates (except the second from the ends).
4 ,, *odd* ordinates (except the end ordinates).

dependent for their value upon the scale to which the body plan is drawn.

A similar course is pursued for each of the water lines to which the calculation is to be made, and is repeated for each angle of the inclination, the readings for each water line at any particular angle of inclination being taken before the body plan is rotated for the next and following angle.

We have therefore the displacement, and the righting arm corresponding to that displacement, relative to each water line at the several angles of heel, from which to construct the cross-curves of stability. If there are buoyant compartments situated above the weather or upper deck, such as a forward hood or forecastle, which will contribute to the stability at large angles of heel, their volumes and moments must be estimated separately, and the resultant moments for any particular angle obtained by combining the results for the main portion as obtained by means of the integrator with those for the hood, etc.

From these cross-curves, the curve of statical stability for any specific displacement and position of centre of gravity may be derived by reading off the ordinate values of the curves at that point on the line of abscissæ indicating that displacement, and correcting for the *actual* position of the centre of gravity of the vessel as already described.

It must be noted that in fixing the positions of the water planes the highest and lowest should be chosen at least to embrace the maximum load and the light draught respectively of the vessel, the intermediate water lines being drawn at equal intervals between them. The number usually selected is five or six at least, so as to give a sufficient number of points to guide us in drawing the cross curves correctly.

Fig. 46 shows the cross-curves of stability relating to the twin-screw passenger steamer referred to on p. 79. These curves are generally typical, but their characteristics will depend entirely on the form of the vessel, the angle at which the deck edge becomes awash, and the vertical position of the assumed centre of gravity, and will therefore vary in character with the type of ship.

The statical stability curves shown in Fig. 42 were constructed from the cross-curves drawn in Fig. 46.

Having explained the principles upon which metacentric curves and curves of statical stability are constructed and the uses to which they individually may be placed, we will proceed to discuss the curves relating to specific types of vessels, and endeavour to derive some information from the study of these curves, as to their distinctive characteristics, and in what way they are affected by the type of vessel to which they refer.

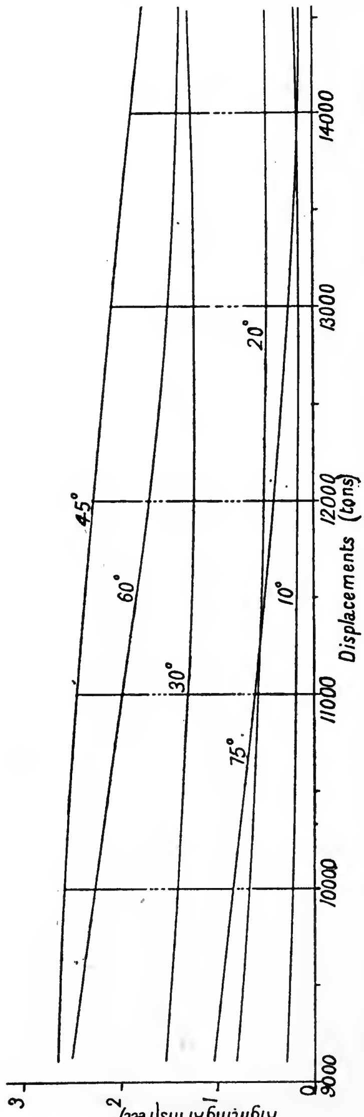


FIG. 46.—Cross-Curves of Stability for First-Class Twin-screw Passenger Steamer referred to on p. 79. C.G. assumed at 24·0 ft. above base.

We shall therefore now give one or two examples of metacentric curves, and first we will take the case of the first-class passenger vessel referred to on p. 79. Fig. 47 shows the metacentric curve for this vessel, and upon the diagram are marked the centres of gravity for the light and load conditions, the particulars of which are as follows :—

(1) Light condition, mean draught 17 ft. 9 in., displacement 9,150 tons.

(2) Load condition, mean draught 25 ft. 9 in., displacement 14,450 tons (bunkers full, 2,000 tons of cargo with stores, fresh water and full complement of passengers on board).

This vessel is of fairly fine form with a displacement coefficient of ·65 and a speed of 18 knots, and is typical of the medium-speed first-class

passenger liner. Such vessels when loaded are of comparatively deep draught relative to beam, with a slowly diminishing water plane area from the load to the light draught. The highest position of the metacentre generally corresponds with the least (the light) draught, and from this point the curve of metacentres falls slowly to a minimum at some point between the light and load draughts, rising again to the maximum (the load) draught, at which the height of metacentre above base is usually less than at the light draught.

Generally there is little (and sometimes negative) metacentric

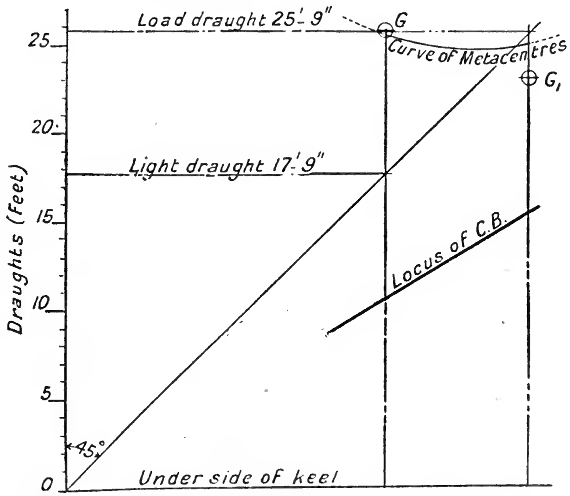


FIG. 47.—Metacentric Diagram for Twin-screw Vessel referred to on p. 79.

height in the light condition, and the curve of metacentres almost invariably lies at about or entirely below the load draught-line, meaning, by the "curve of metacentres," that portion of the curve corresponding to the draughts intermediate to the light and load conditions.

There is a similarity between this curve and that shown in Fig. 48, referring to a modern steam yacht, of fine form and coefficient of displacement ·57. In this latter case, however, the curve is flatter and lies near, but entirely above, the load draught line, and is such that there is a substantial amount of metacentric height in the light condition.

These particulars may be compared with the diagram

(Fig. 49) referring to a modern cross-Channel paddle steamer for passenger service. The nature or form of the curve is not unlike those shown in the Figs. 47 and 48. It, however, descends rather quickly from the light to the load draught, reaching its lowest point at a draught considerably greater than the latter. This is accounted for, apart from the difference in form of the vessels, by the small margin of displacement between these limiting draughts amounting to some 150 tons only—the equivalent of passengers, stores and fuel.

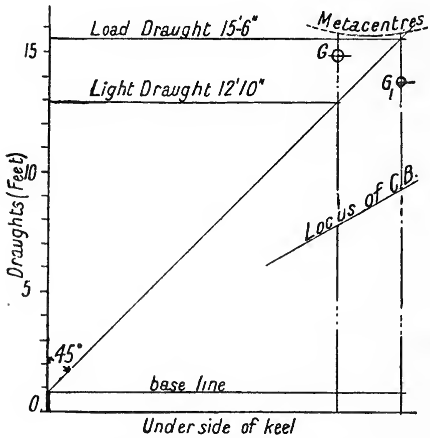


Fig. 48.—Metacentric Diagram for Steam Yacht referred to on p. 87.

The metacentric curve in this class of vessel lies well above the load water line, an effect produced by the comparatively shallow draught to which these vessels are designed, with their attendant augmented area and moment of inertia of water plane.

A comparison between particulars relating to the three types of steamer just considered is set out in the following table :—

Ratio :—

	T.S. Passenger Steamer.	T.S. Steam Yacht.	Paddle Steamer.
$\dfrac{\text{Height of centre of buoyancy above base}}{\text{draught*}}$	= ·594	·574	·562
Coefficient of displacement	= ·65	·57	·544
Coefficient transverse moment of inertia of water plane area.	= ·625	·568	·518

where the moment of inertia coefficient =

$$\frac{\text{Actual transverse moment of inertia of water plane}}{\frac{1}{12} \times \text{length} \times (\text{ext. breadth})^3}$$

* Where a bar keel is fitted, as in the case of the yacht and paddle steamer, the draught, for the purpose of this ratio, is taken as moulded draught + thickness of garboard strake ; and the baseline in each case is taken at zero draught.

It will be seen by comparing the coefficient of moment of inertia of water plane area with that for displacement in the three cases that we would anticipate a similarity in the metacentric diagrams for the twin-screw vessels, while the metacentric curve in the case of the paddle steamer would be expected to stand relatively much higher with regard to the base-line. This is borne out by the diagram.

It will be observed that the relative positions of the centre of gravity in the light and loaded conditions are reversed in the case of the paddle steamer as compared with the twin-screw passenger liner and the yacht, due to the fact that in this case the bulk of the extra weight (that of the passengers) is carried high up at the level of the upper or spar deck, thus raising the centre of gravity of the vessel; whereas the weights which contribute to the loading in the cases of the twin-screw vessels are relatively carried low down, the position of the centre of gravity falling in consequence.

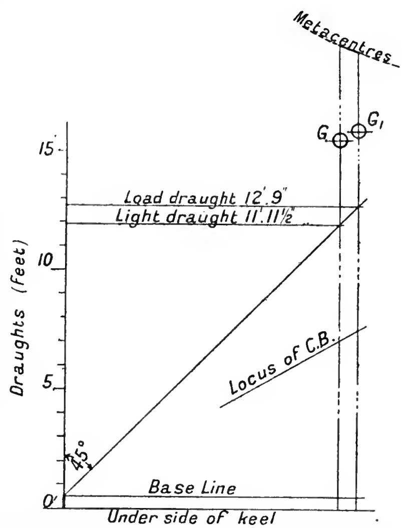


Fig. 49.—Metacentric Diagram for Paddle Steamer referred to on p. 88.

The metacentric curve in the case of warships closely resembles that shown in Fig. 49 for the paddle steamer. The highest position of the metacentre usually corresponds to the light draught, and descends to a minimum at a draught which would be considerably greater than the maximum or load draught for the seagoing or fully-equipped condition. Moreover, the curve lies entirely above the load line, and is generally situated relatively much higher in the case of the small swift torpedo-boats and torpedo-boat destroyers than for the larger vessels of the cruiser or battleship class.

There is generally very little variation in the value of GM, or metacentric height, between the light and load conditions, due to the consumable stores in warships being situated at

about the height of the centre of gravity for the light condition but mostly *below* that position, so that as the vessel lightens the centre of gravity rises, until the position for the light condition is reached when all stores, etc., are consumed. Where such vessels have upper coal bunkers, from which the coal requires to be trimmed into the lower bunkers from time to time, the centre of gravity may at first fall below the position for the load condition and then rise again as the light condition

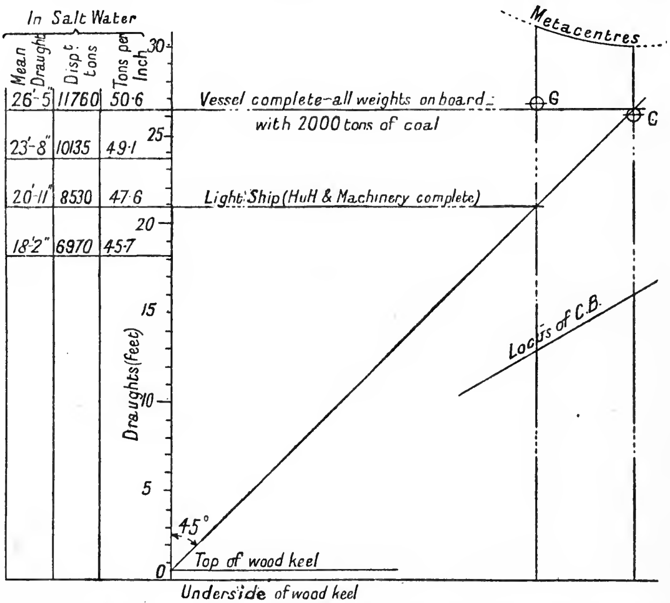


FIG. 50.—Metacentric Diagram for First-Class Cruiser (wood sheathed).

is approached; but should the upper bunkers remain intact while the coal in the lower bunkers only is consumed, the centre of gravity might rise from the position for the load condition to a height greater than that for the light condition before it begins to fall as the coal in the upper bunker is consumed.

Fig. 50 is the metacentric diagram for a first-class wood-sheathed cruiser, while Fig. 51 refers to a 30-knot torpedo-boat destroyer, the metacentric curve for which has a much steeper descent from the light to the load draught than in the case of the cruiser; the metacentre relatively stands much higher

and the value of GM is proportionately much greater, thereby insuring the quality of stiffness and preserving the upright

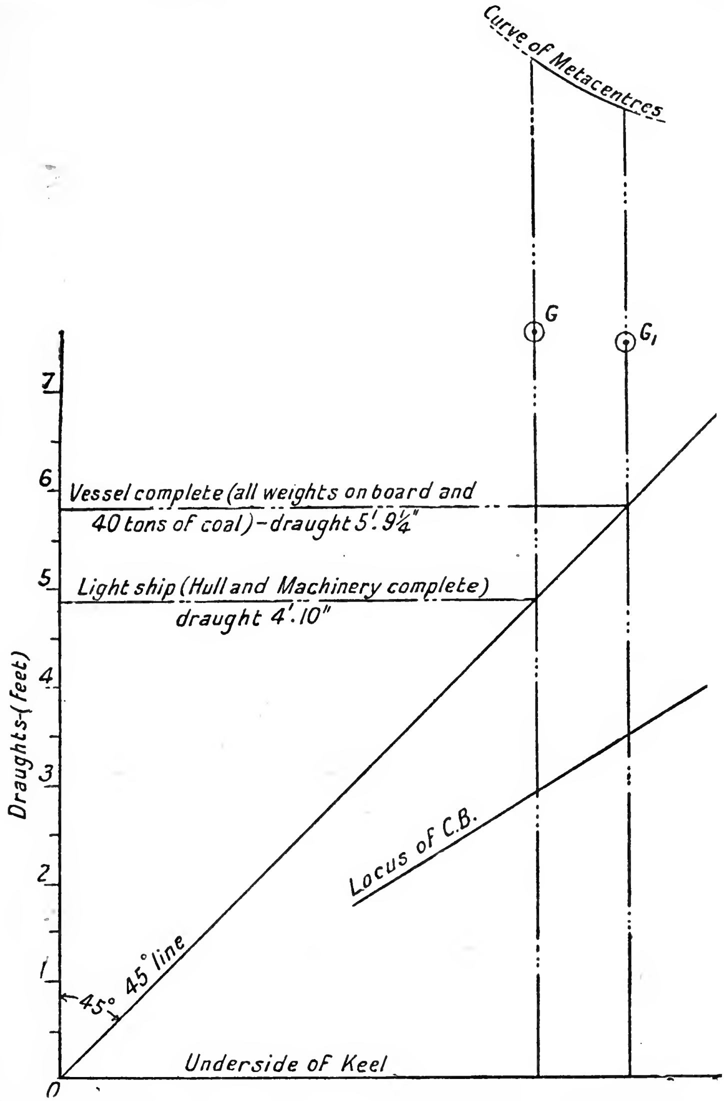


FIG. 51.—Metacentric Diagram for 30-knot Torpedo-boat Destroyer.

position of the vessel under the action of external heeling forces. Reference was made on p. 88 to "Coefficients of

Displacement" and "Moments of Inertia" of water plane area, and it was there seen how these coefficients varied with the type of vessel. The area and contour of the water plane naturally depends upon the form given to the under-water portion of the vessel, and *vice versâ*. The particular form given to the vessel will depend upon the service for which she is to be employed; for vessels intended purely for the carrying trades where speed is not the primary feature, but where, on the other hand, good carrying capacity is essential, the lines are full, and in not a few instances such vessels are little less than rectangular parallelopipedons with the angles eased away for sightliness and convenience in handling.

As we proceed to speeds beyond some 9 or 10 knots the form has to be modified to meet the new demands as regards fluid resistance, etc.

Such modification of form means a reduction of displaced volume from that of the rectangular vessel, represented by the product

$$\text{Length} \times \text{breadth} \times \text{mean draught},$$

to that represented by the product

$$k\,(\text{Length} \times \text{breadth} \times \text{mean draught}),$$

where k is a coefficient the value of which depends upon the type of vessel under consideration.

Some particulars relating to certain classes of vessel of modern construction are given in the table on p. 93 :—

Simultaneously with a variation in displacement there will be a corresponding change in the value of area and moment of inertia of water plane. For the rectangular vessel the transverse moment of inertia would be $\frac{1}{12}\cdot L\cdot B^3$, but if we choose a coefficient k_1, such that $k_1\,(\frac{1}{12}\cdot L\cdot B^3)$ is the moment of inertia of water plane area for the particular vessel, this coefficient will vary generally according to the figures given in the table. It must be borne in mind that these figures may only be used to give *first* approximations in the case of a vessel the principal dimensions of which are known or assumed, and cannot be relied upon to give the accurate results required in the ultimate design. In all cases the detailed calculations must be methodically worked out from the lines of the vessel finally decided upon.

TABLE GIVING PARTICULARS OF COEFFICIENTS OF DISPLACEMENT, ETC., IN TYPICAL CASES.

	Steam Yachts.		1st Class Paddle Steamers.		1st Class Passenger Vessels.				Cruisers and Battleships.		Torpedo Boat Destroyers.		Cargo Vessels (tramps).			
	a.	*b.*	*a.*	*b.*	*a.*	*b.*	*c.*	*d.*	*a.*	*b.*	*a.*	*b.*	*a.*	*b.*	*c.*	*d.*
Length of vessel (feet) ...	270·0	275·0	240·0	360·0	490·0	350·0	270·0	500·0	360·0	450·0	195·0	210·0	285·0	250·0	270·0	445·0
*Load draught (feet) ...	14·75	13·5	7·2	12·0	26·0	14·8	10·0	24·0	19·75	24·25	5·6	5·75	18·75	16·25	18·3	25·12
Speed in knots	15·5	20·0	18·5	21·0	17·0	19·0	19·5	21·0	19·5	20·5	27·0	30·0	9·0	11·0	11·5	11·75
Coefficient of displacement.	·572	·500	·540	·545	·645	·565	·580	·590	·525	·535	·445	·475	·800	·730	·704	·750
Coefficient of transverse moment inertia of water plane.	·565	·472	·510	·520	·623	·505	·463	·535	·470	·467	·435	·465	·768	·620	·711	·700
Coefficient of longitudinal moment inertia of water plane.	·470	·395	·407	·420	·545	·425	·390	·465	·405	·40	·390	·400	·757	·573	·590	·609
Height of longitudinal metacentre above centre of buoyancy (feet).	340	370	495	670	650	535	385	650	450	560	490	550	335	260	282	540

* Mean draughts taken to the designed water lines. In vessels with bar or wood keels the depth of the latter is excluded in determining the coefficients of displacement.

Such coefficients, however, will be found extremely useful in the early stages of a design, before the principal dimensions are actually fixed, in enabling the designer to estimate approximately the required particulars to guide him in scheming out the design.

In conjunction with these coefficients it is advantageous to have a ready method of approximating to the vertical position of the centre of buoyancy. A good approximate rule by which the depth (H) of the centre of buoyancy below the water plane may be determined is that given by Mr. Furze Morrish,* viz. :—

$$H = \frac{d}{3} + \frac{D}{6}$$

where d = volume of displacement $\div$ area of water plane.
and D = mean draught (without bar keel if any).

Turning from the simple metacentric diagram, we will now consider the curves of statical stability relating to the several types of vessel already selected and see how the nature of the curves differs (if at all) the one from the other ; in other words, to see how the type of vessel affects the form of the stability curve, and what influence a modification of freeboard, beam, position of centre of buoyancy and centre of gravity, etc., will exert on the value of righting arm and the range of stability.

The simple prismatic vessel of rectangular shape has been considered (p. 62), and we learned there that the general influence of freeboard is much greater than that of beam in augmenting the righting arm and lengthening out the curves of stability.

These characteristics will be found generally to be maintained in the case of bodies of ship-shape form.

The statical stability curves for the twin-screw first-class passenger vessel referred to on p. 79, are shown in Fig. 42 for the conditions of loading there stated.

In comparing these curves it is obviously only possible, in any one vessel, to analyse the effect of freeboard and the position of the centre of gravity, since the dimensions of the vessel remain constant. The freeboard increases or decreases

* See Transactions I. N. A., 1892.

with a diminution or increase of draught respectively, and the position of the centre of gravity—dependent as it is upon the distribution of the weights which go to make up the freight—will generally change with it.

Assuming curve 1 (light condition) as the standard with which to make comparison, we will select any one of the remaining curves, say No. 6; the leading particulars under these conditions are at follows :—

	Draught.	Displacement.	Centre of* Buoyancy.	Centre of* Gravity.	Freeboard.	Beam.	G.M.
	ft. ins.	tons	ft.	ft.	ft.		ft.
Curve No. 1 . .	17 9	9,150	10·74	25·8	21·0	B.	– ·24
Curve No. 6 . .	25 9	14,450	15·63	23·08	13·0	B.	2·04

* Above base.

The vessel with low freeboard (curve 6) and a diminished height of centre of gravity has an increase of metacentric height = 2·28 ft., and, as naturally would be expected, an increased righting arm; this is borne out by the curves. The angle of inclination at which the righting arm begins to diminish will depend upon the angle at which the deck edge becomes awash—36° for the light and 24° for the load condition; and the downward slope of the curve will depend upon this also. By reference to Fig. 42, it will be seen that the rate of diminution of righting arm is practically the same in the two cases, with an increase of range for the loaded condition equal to about 15°.

Had the centre of gravity remained fixed or practically fixed in the two cases the character of the curve for the loaded condition would have been considerably modified; indeed, the curve would have fallen completely within curve 1, as shown by curve A, Fig. 42, and the vessel under such a condition would have been actually unstable until an angle of about 22° of inclination had been reached, beyond which she would have possessed stability only for a further inclination of 35°.

To analyse generally the effect of changes in the value of beam, freeboard, position of centre of gravity, etc., upon curves of stability, comparison will now be made with relation to vessels of different but representative types and dimensions. For this purpose the diagrams (Figs. 52 and 53), have been constructed to include the curves relating to the several vessels already considered; of these, Fig. 52 relates to the light and

Fig. 53 to the load conditions of the vessels. The curves shown on the diagrams are quite sufficient for our purpose of comparison, and embrace practically all the types of vessel likely to come within the scope of the designing office.

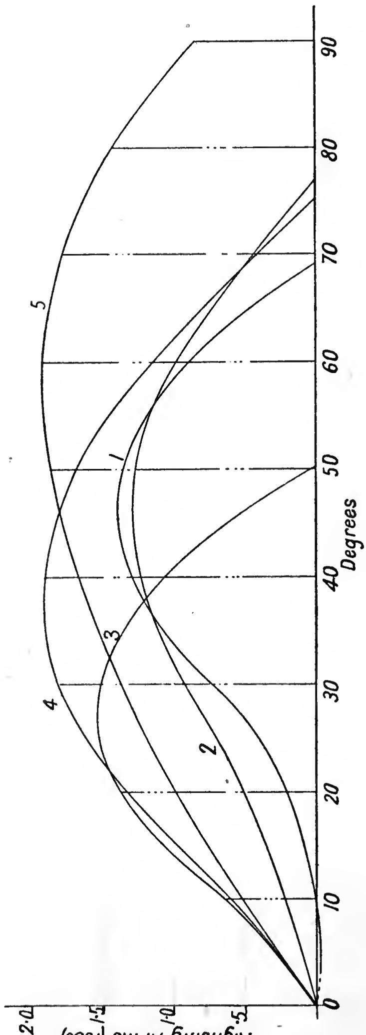


FIG. 52.

It may be stated that in each case the position of the centre of gravity has been carefully ascertained by subjecting the vessel to an inclining experiment, and the ultimate position of the centre of gravity for the conditions to which the curves refer has been deduced therefrom by calculation from the known amount and position or distribution of the weights contributing to the displacement. It is assumed that no shifting of the weights would occur as the vessel inclines from the upright; that there is no loose water in the vessel, and consequently that all compartments used for holding fluids, such, for example, as water ballast tanks, are either completely full or entirely empty; and that the buoyancy and reserve buoyancy is not interfered with in any way by water entering the vessel as she inclines from the upright.

This is the ideal condition which, although perhaps it might not be quite fulfilled in practice, yet is nearly approached for ordinary inclinations to which the vessel may be subjected; and even at extreme

angles of inclination water may be prevented from entering the interior of the vessel by battening down hatches and closing openings in the deck, while cargo and weights generally may be prevented from shifting by properly trimming and securing them prior to the vessel proceeding to sea. It is under such conditions only that the vessel will be able to heel to such extreme angles from the upright as are illustrated by the range of stability depicted in these diagrams before losing stability altogether. It is probable however, that should the vessel at any time approach such extreme angles of inclination she would founder, due to the inertia of the rotating mass tending to carry her beyond the angle at which the stability vanishes, or from some other cause, such as the shifting of weights or ingress of water.

PARTICULARS OF DISPLACEMENT, ETC., FOR THE VESSELS TO WHICH FIGS. 52 AND 53 REFER.

		Displacement.	Draught.	Beam.	Depth (Md.).	C.G. above base.	G.M.
		tons.				ft.	ft.
Curve 1. T.S. Passenger Vessel	Light.	9150	17′ 9″	59′ 0″	38′ 9″	25·8	-·24
	Load.	14450	25′ 9″			23·08	2·04
Curve 2. T.S. Yacht	Light.	1720	12′ 10″	34′ 0″	21′ 3″	14·0	·98
	Load.	2230	15′ 6″			12·85	2·15
Curve 3. Paddle Steamer (Passenger)	Light.	2630	11′ 11½″	42′ 0″	25′ 6″	14·96	3·89
	Load.	2860	12′ 9″			15·30	3·17
Curve 4. 1st Class Cruiser.	Light.	8530	20′ 11″	68′ 2″	39′ 9″	26·74	4·11
	Load.	11760	26′ 5″			26·34	3·44
Curve 5. T.B. Destroyer (30 knots)	Light.	265	4′ 10″	21′ 0″	12′ 9″	7·56	2·80
	Load.	340	5′ 9¼″			7·40	2·40

Comparing the corresponding curves for the mercantile vessels, it will be seen that in each case, except that of the passenger paddle steamer, not only is the righting arm at any particular angle of inclination increased, but the range of stability also, as the vessel passes from the light to the load condition. This is ordinarily the case, and is due to the fact that generally the reduction in height of the position of centre of gravity relative to the baseline is greater than the negative effect produced by the reduction in freeboard. The fall of the metacentre consequent on the increased displacement of the vessel is also usually less than that of the centre of gravity, so that the value of GM is increased.

With reference to the paddle steamer (curves 3) the case is reversed, and is explained by reason of the weights (other

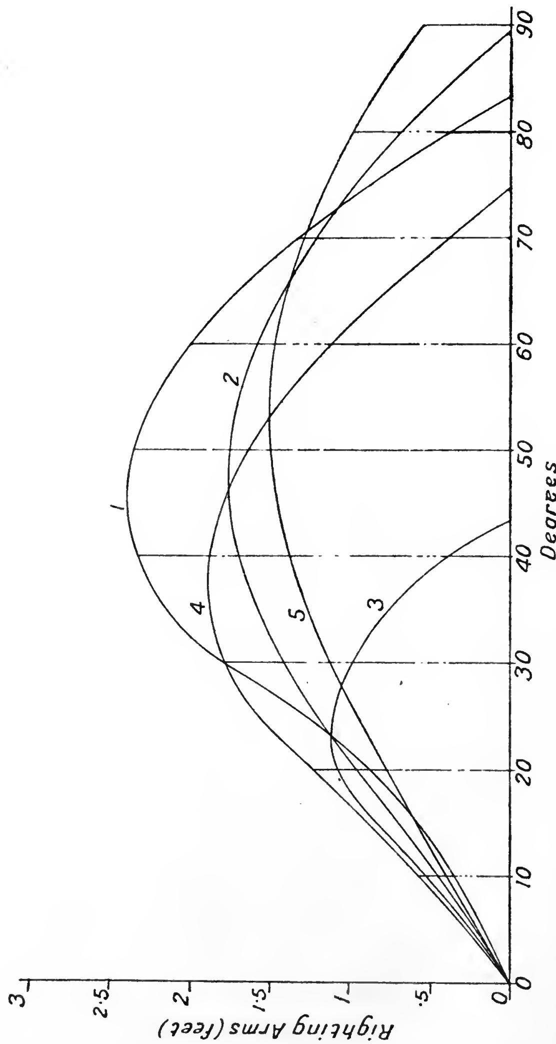


FIG. 53.

than fuel), viz., passengers, stores, and provisions, which go to make up the loading, being situated high up in the vessel, in consequence of which the centre of gravity for the load condition is raised and the horizontal distance between the

centre of gravity and centre of bouyancy at any particular angle of heel is reduced—*i.e.*, there is a reduction in righting arm, combined with which the diminished freeboard has the effect of shortening the range, so that the stability curve for the load condition would fall within that representing the light condition.

The relation between the stability curves for the light and load conditions in the case of the cruiser and the torpedo-boat destroyers is somewhat similar to that of the paddle steamer, but due to another cause.

With the war-vessels the position of the centre of gravity is higher in the light than in the load condition for the reasons stated on p. 89, although the difference generally is not great. The metacentric curves have been seen to be much steeper at the point corresponding to the light draught than in the case with the mercantile vessels, and this is specially so with the torpedo-boat destroyer.

On this account the initial stability is less in the load than in the light condition, and generally the result is a decrease in the value of the righting arm, at least for a considerable angle of inclination, with little difference in the ultimate range of stability.

A careful study of Figs. 52 and 53 will be found to be profitable, inasmuch as the curves illustrate in a comparative sense the effect on stability of freeboard, beam, and position of centre of gravity in vessels of widely different form and dimensions.

A final reference may be made to Fig. 54 as illustrating in a direct way the effect of beam and freeboard on the character of stability curves in one specific case of a vessel, the transverse dimensions of which have been modified for the purpose, much in the same way as was done in the case of the prismatic rectangular vessel (p. 70).

Curve 1 refers to the normal vessel of 50 ft. beam, moulded depth 28 ft., freeboard 6·5 ft., and initial stability 2·6 ft.

Curves 2 and 3 represent the vessel with an increase of beam of 5·0 ft. and 10·0 ft. respectively, the depth and freeboard remaining unchanged.

Curves 2_1 and 3_1 show the stability conditions of the same vessel in which the beam is the same as in the normal vessel,

but the depth, and with it the freeboard, have been increased by 4·5 ft. and 9·0 ft. respectively.

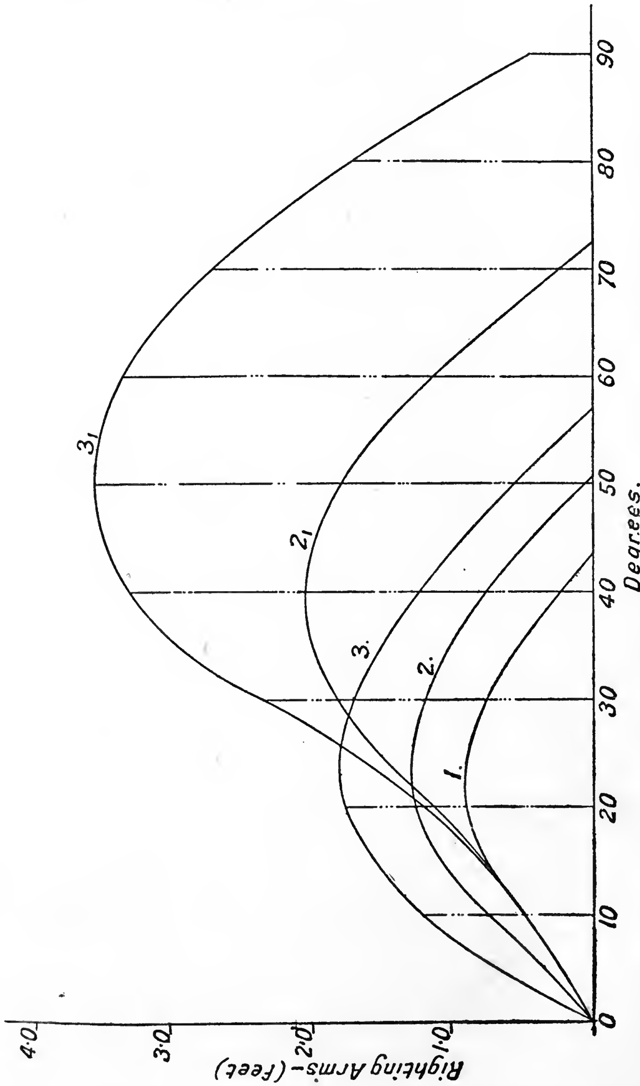


FIG. 54.—Diagram illustrating the effect on Statical Stability of increasing the Freeboard and Beam.

Curve 1, of statical stability of normal vessel
" 2, " " " " with beam increased 5·0 ft.
" 3, " " " " " " 10·0 "
" 2_1, " " " " freeboard increased 4·5 "
" 3_1, " " " " " " 9·0 "

Throughout the draught remains constant, viz., 21·5 ft., with a constant height of centre of gravity above base, so that, whereas in cases 2 and 3 the initial stability has undergone a

change, the initial stability in cases 2_1 and 3_1 remains the same as with the normal vessel.

The increase in value of GM in cases 2 and 3 was 1·55 ft. and 4·2 ft. respectively.

It will be observed that the effect of increasing the beam is to augment the value of righting arm and increase the range of stability, but as the angle at which the deck edge becomes awash is very little smaller than in the case of the normal vessel, the righting arm reaches its maximum value at an angle of inclination which is practically the same in each case.

With reference to the effect of increase of freeboard, it will be seen that there is coincidence of the curves 2_1 and 3_1 with curve 1 until the angle of inclination of 15° is reached, at which the deck edge of the normal vessel becomes awash; thereafter the righting arms are considerably increased, the maximum values being reached at angles of heel much greater than in the case of the normal vessel, or of the vessel modified in respect of increase of beam, the angles of vanishing stability being also greatly augmented.

So far our remarks have been confined to statical stability in the transverse or athwartship direction, but there is another kind, termed "Longitudinal Stability," which must now be considered. It is the stability of a vessel when inclined in the fore-and-aft or longitudinal direction from her normal position of equilibrium.

The principles which have been applied to transverse inclination are equally applicable to angular movements in the longitudinal direction, and in respect of the latter are chiefly of service in determining the trim and change of trim of a vessel under any specific condition of loading, brought about by the manner in which the weights constituting the load are distributed longitudinally.

By "trim" is meant the "difference between the draughts forward and aft"; and by "change of trim," the alteration produced in that "difference of draughts" by :—

(*a*) Moving weights already on board, in the longitudinal direction;

(*b*) Placing weights on board;

(*c*) Removing weights already on board.

The exact amount of trim or change of trim is determined

by making use of the position of the metacentre calculated for longitudinal inclination, termed the "Longitudinal Metacentre," in a manner similar to that already explained when describing the inclining experiment on p. 74.

The formula given on p. 25 for the height of metacentre above the centre of buoyancy is true for inclinations in any direction, whether transverse, longitudinal, or any skew direction intermediate to these, so long as we remember that the moment of inertia of water plane area is taken about an axis through the centre of area of the water plane perpendicular to the particular plane of rotation. In the case of rotation or inclination in the longitudinal direction the axis will be perpendicular to the middle line plane of the ship.

Very little consideration will suffice to show that the moment of inertia of water plane for longitudinal inclination will, generally, be considerably greater than that for inclination in the transverse direction, being represented by some function of L^3B and LB^3 respectively, where L denotes the length and B the beam of the ship.

If we take k_2L^3B as representing the longitudinal moment of inertia of water plane area, then k_2 is a coefficient, which will vary with the type of ship; particulars of this coefficient are given in the table on p. 93.

For ordinary vessels, if the mean draught be approximately ·06 of the length of the vessel, the height of the longitudinal metacentre above the centre of buoyancy may be taken *roughly* as equal to the length of the vessel; this, however, should not be used for purposes of calculation where results are required to even only an approximate degree of accuracy, and the statement is only made here to illustrate the discrepancy generally existing between the height of metacentre above the centre of buoyancy when referred to the longitudinal and transverse directions of inclination; to the better and faster class of vessel it has no application.

To illustrate the relative value of the "height of longitudinal metacentre above the centre of buoyancy" and "length of the vessel," particulars have been included in the table on p. 93 for the several types of vessel already considered. It was shown when dealing with inclinations in the transverse direction that the height of metacentre above the centre of

buoyancy generally increased with a decrease in draught on account of the rate of diminution in the value of moment of inertia of water plane area being less than that of the displacement of the vessel.

The same argument applies, but with greater emphasis, to inclinations in the longitudinal direction, and for this reason it is useful to construct curves of "longitudinal metacentric height," or rather of " height of longitudinal metacentre above centre of buoyancy," for merchant vessels where the exigencies of trade admit of big discrepancies in draught and displacement.

In the case of warships, yachts, and vessels engaged in passenger service to the exclusion of large cargo capacity, where at any time there is no great variation in draught, such curves are not of so great a value, and in no case is it considered necessary to construct curves of statical stability for longitudinal inclinations.

The curve of longitudinal metacentres for the first-class twin-screw passenger vessel referred to on p. 79, together with other curves, is shown in Fig. 56.

It is clear that the determination of the amount of trim from which to derive the draught forward and aft forms a very necessary item of the calculation in ship designing. Vessels which are intended for certain routes and to engage in specific trades are often limited to a maximum draught in order that navigation may be possible ; this is due to the shallowness of the water at certain points of the route not admitting of more than a certain draught, and the knowledge of how to alter the trim of a vessel by moving weights on board through certain distances so as to reduce the draught at which the vessel may be floating is often useful in enabling the vessel to pass over a bar at the entrance to a river, the sill at a dock entrance, or to negotiate certain channels. Apart from these conditions, vessels are usually required to float with a greater draught aft than forward, and at the lighter displacements this is necessary in order that the propellers may be sufficiently immersed. It is therefore absolutely necessary to be able to determine the trim of a vessel.

An increase of draught forward or aft may be brought about by moving weights already on board in the direction of the bow or stern, as the case may be, or by placing weights on

board at positions towards the bow or stern, to be presently determined ; and the amount by which the draught is altered will depend upon the amount of the weights moved and the positions which they finally take up in the longitudinal direction.

Suppose a vessel (Fig. 55) floating originally at the water line WL. Let M_2 represent the longitudinal metacentre, H the centre of buoyancy, and G the centre of gravity, so that HG and M_2 are in the same vertical line.

Now suppose a weight w, already on board and situated at a, to be moved aft to the position b through a distance x.

The vessel will suffer a small angular movement θ in the

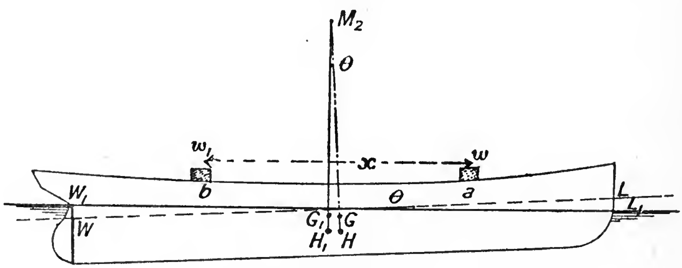


Fig. 55.

longitudinal direction, so that the water line changes to W_1L_1 and the centres of buoyancy and gravity to H_1 and G_1 respectively. The line H_1G_1 will now be vertical and will pass through M_2, the angular movement being supposed small, and GG_1 will be parallel to ab.

Now generally ab will be approximately parallel to the water plane, in which case GG_1 may be taken as being perpendicular to GM_2 or G_1M_2.

We have :—

$$GG_1 = \frac{w \times x}{W} = GM_2 \tan \theta,$$

where W is the weight of water displaced ;

$$\therefore \tan \theta = \frac{w \times x}{W \times GM_2}.$$

For small inclinations we may take WL or W_1L_1 as equal to the length of the vessel, and if we take y to represent the change of trim we have :—

$$y = WW_1 + LL_1,$$

$$\text{but } \tan\theta = \frac{WW_1 + LL_1}{WL} = \frac{y}{L},$$

where L = length of vessel on the water line,

$$\therefore\ y = L\tan\theta = \frac{w \times x \times L}{W \times GM_2} = \text{change of trim.}$$

If the change of trim were one inch, *i.e.*, $\frac{1}{12}$ of a foot, we should have :—

$$w \times x = \frac{W \times GM_2}{12L},$$

where $w \times x$ is termed "the moment to change trim one inch" in foot-tons if w be expressed in tons and x in feet.

This formula is used to construct a curve of "moments to

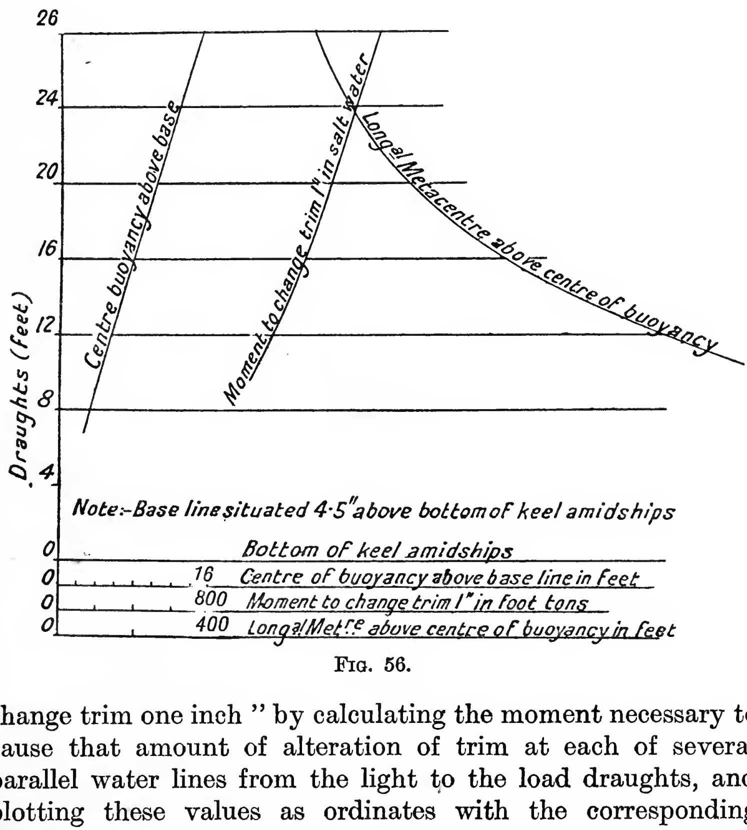


FIG. 56.

change trim one inch" by calculating the moment necessary to cause that amount of alteration of trim at each of several parallel water lines from the light to the load draughts, and plotting these values as ordinates with the corresponding draughts as abscissæ. Such a curve is drawn in Fig. 56, in

which is also shown the curves of "centre of buoyancy above base," and "longitudinal metacentres above centre of buoyancy."

Generally it may be assumed that GM_2 is *much greater* than HG, and without appreciable error may be taken equal to HM_2.

Referring to Fig. 55, it will be seen that the draught aft has been increased by WW_1, and the forward draught diminished by LL_1, by virtue of the moment $w \times x$.

If the water plane area were such that its centre of gravity lies at (or nearly at) the mid-length of the vessel, then for small angles of longitudinal inclination the line of intersection of the water planes WL and W_1L_1 may be assumed to be situated at

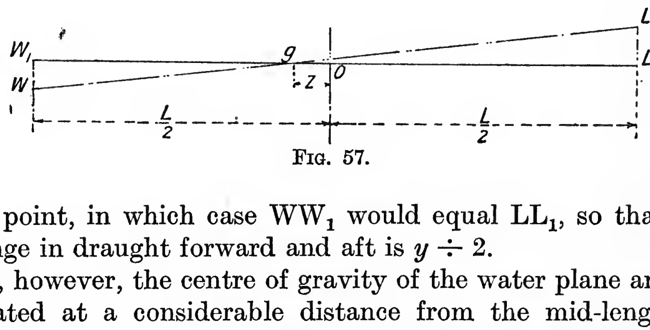


FIG. 57.

this point, in which case WW_1 would equal LL_1, so that the change in draught forward and aft is $y \div 2$.

If, however, the centre of gravity of the water plane area be situated at a considerable distance from the mid-length as compared with the half-length of the vessel, the change of draught forward and aft would not be equal and would have to be considered as follows :—

Let o and g, Fig. 57, represent the mid-length and centre of gravity of water plane area respectively, then :—

$$\frac{WW_1}{WW_1 + LL_1} = \frac{WW_1}{y} = \frac{Wg}{L} = \frac{\frac{L}{2} - Z}{L} = \frac{1}{2} - \frac{Z}{L}$$

i.e.,

$$WW_1 = y\left(\frac{1}{2} - \frac{Z}{L}\right)$$

and similarly

$$LL_1 = y\left(\frac{1}{2} + \frac{Z}{L}\right)$$

give the change of draught at the after and forward end of the vessel respectively : y would be determined from the curve of moments to change trim one inch.

If the change of trim be brought about by *placing* weights

on board, the determination of the amount will depend upon whether the weights are only moderate or of comparatively large value.

If moderate, the weights are first assumed to be placed vertically over the centre of gravity of water plane area, in which case the vessel will be additionally immersed bodily, and the newly assumed water plane will be parallel to the initial water plane. The weights may now be moved to the positions assigned to them, and the moment causing change of trim will be the algebraic sum of the products of each weight and the distance through which it is moved relative to the centre of gravity of the water plane, taking movements forward to be of opposite sign to movements aft. Change of

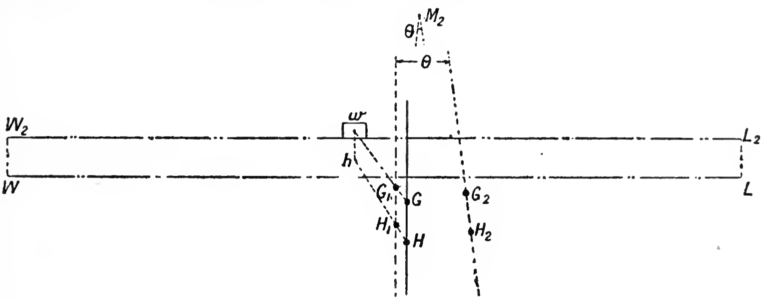


Fig. 58.

mean draught as well as change of trim will take place, but if the weights be moderate the former will be small.

For weights of considerable magnitude there will be an appreciable change in mean draught accompanying change of trim, which latter may or may not be large.

First assume the change of trim to be small, and consider the vessel to sink bodily, so that the new displacement exceeds the initial by the amount of the weights placed on board, and that the new and the initial water planes are parallel. The position of the new water plane may be easily calculated from the lines* of the vessel and the centre of buoyancy, h, of the added volume determined.

The added weights w are then assumed to be placed vertically over h—or, more correctly, in the same vertical line with it—and afterwards moved individually into their assigned positions

* See footnote on p. 108.

and the algebraic sum of their moments $w \times d$ (say) determined as in the previous case. The change of trim may now be calculated from the formula :—

$$y = \frac{w.d.L}{(W + w)\ GM_2}$$

where the height of metacentre M_2 above centre of buoyancy H_1 has been newly calculated from the lines* to the new water plane W_2L_2.

Second, assume the change of trim to be large while the angle θ of inclination in the longitudinal direction remains small. The curve of "moments to change trim one inch" construc ted on the assumption of small changes of trim, cannot now be used. Taking G_1G_2 as practically equal to H_1H_2, we have $\theta = \tan^{-1} \dfrac{w.d}{(W + w)\ G_1M_2}$ from which to determine the position of the actual water plane ; this is obtained tentatively.

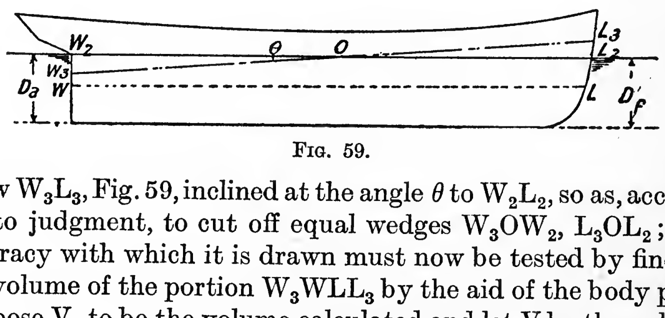


Fig. 59.

Draw W_3L_3, Fig. 59, inclined at the angle θ to W_2L_2, so as, according to judgment, to cut off equal wedges W_3OW_2, L_3OL_2 ; the accuracy with which it is drawn must now be tested by finding the volume of the portion W_3WLL_3 by the aid of the body plan. Suppose V_1 to be the volume calculated and let V be the volume ($w \times 35$) required. If A be the area of the water plane W_3L_3, then $\dfrac{V_1 - V}{A}$ is the amount by which W_3L_3 must be moved up or down parallel to itself to obtain the correct displacement, from which the corrected draughts are obtained as follows :—

$$\text{Corrected draught aft} = D_a - \left(W_2W_3 \mp \frac{V_1 - V}{A}\right).$$

$$\text{,, ,, forward} = D_f + \left(L_2L_3 \pm \frac{V_1 - V}{A}\right).$$

If the weights w were of such magnitude that the angle θ were

* If the water plane is parallel to those to which the displacement scale or diagram is constructed, then the latter may be used.

considerable, then it could not be assumed that H_2G_2 produced would pass through the metacentre corresponding to the water line W_2L_2, and $\tan\theta$ would not be equal to $\dfrac{w.d}{(W + w)\, G_1M_2}$. In this case, in order to determine θ, it would be necessary to adopt the principle of cross-curves of longitudinal stability, much in the same way as would be done in the case of transverse inclination.

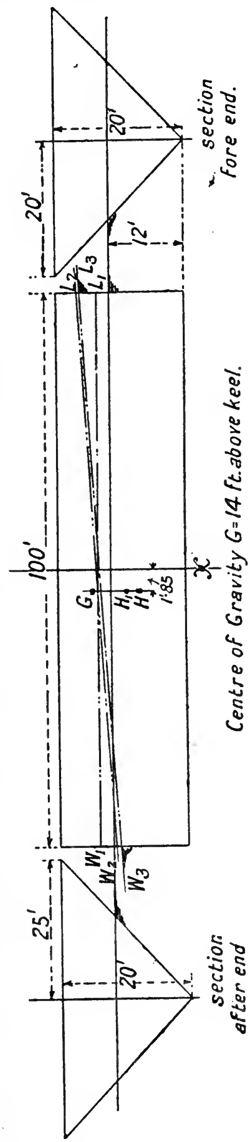


FIG. 59A.

By way of illustration we will take the case of a vessel of varying triangular section, and of the dimensions indicated in Fig. 59A, floating on a level keel at 12 ft. draught. The displacement at this draught is 463 tons and the centre of buoyancy 1·85 ft. abaft amidships.

The vessel is brought down by the head by means of a weight of 150 tons placed in such a position that its centre of gravity is situated 20 ft. before the initial centre of buoyancy and 13 ft. above the keel.

Suppose the weight is placed on board and the vessel sinks bodily, the new and old water lines being parallel.

The new displacement is 613 tons and the new draught 13·82 ft. = 13 ft. $9\frac{7}{8}$ in.

The new centre of buoyancy is 9·21 ft. above the keel and remains unchanged in the fore-and-aft direction, the centre of buoyancy of the added volume of displacement being in the same vertical plane with that for the vessel.

The weight of 150 tons, after being placed vertically over the centre of buoyancy of the new volume of displacement, is moved forward to its assigned position.

The final centre of gravity of the vessel is :—

$$\frac{463 \times 14 + 150 \times 13}{613} = 13{\cdot}8 \text{ ft. above keel.}$$

The moment causing change of trim = 150 × 20 foot-tons.

The height of metacentre above new centre of buoyancy H_1 is :—

$$\frac{2{,}495{,}800}{613 \times 35} = 116{\cdot}33 \text{ ft., } i.e., 125{\cdot}54 \text{ ft. above keel,}$$

or the metacentric height = 125·54 — 13·8 = 111·74 ft.

Therefore longitudinal inclination of the vessel = θ

$$= \tan^{-1} \frac{150 \times 20}{613 \times 111{\cdot}74} = 2^\circ\ 31'.$$

If we draw through the centre of area of the water plane W_1L_1 a line W_2L_2 inclined at angle θ to WL, the draughts become :—

Forward 16·09 ft. = 16 ft. 1 in.
Aft 11·71 ft. = 11 ft. $8\frac{1}{2}$ in.

If we now calculate the displacement of the vessel to the

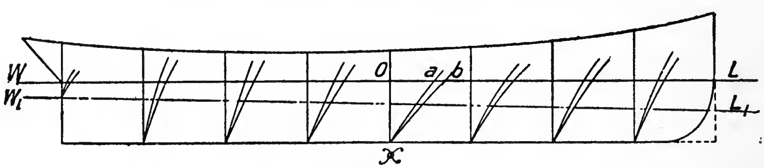


Fig. 59b.

line W_2L_2 we find that it is 208 cubic ft. too great. The area of the water plane W_2L_2 is 3,108 square ft.

Therefore the correct water plane should be situated

$$\frac{208}{3{,}108} \times 12 = \tfrac{3}{4} \text{ in. below } W_2L_2.$$

This is shown by the line W_3L_3 in the figure.

The corrected draughts become :—

Forward (16 ft. 1 in.) — $\frac{3}{4}$ in. = 16 ft. $0\frac{1}{4}$ in.
Aft (11 ft. $8\frac{1}{2}$ in.) — $\frac{3}{4}$ in. = 11 ft. $7\frac{3}{4}$ in.

It was stated on p. 108 that where the longitudinal inclination of the vessel is considerable the metacentric method of computing the trim or change of trim would not apply, and that the *principle* of cross-curves of longitudinal stability must in such a case be resorted to.

For this purpose the "Bonjean" curves are useful. These

consist in drawing at certain intervals along the profile, curves of areas and moments of those areas above the base or keel line, of the transverse sections of the vessel, the ordinates of the curves being set out perpendicular to the line representing the particular section. *E.g.*, *a* and *b* in Fig. 59B represent respectively two such curves relating to the midship section, *oa* denoting the area and *ob* the moment of the area of midship section up to the water line WL. By the aid of such curves we may obtain the volume of displacement and the position of the centre of buoyancy with respect to the keel and (say) the midship section, to any water line W_1L_1, the inclination of which to the keel is θ.

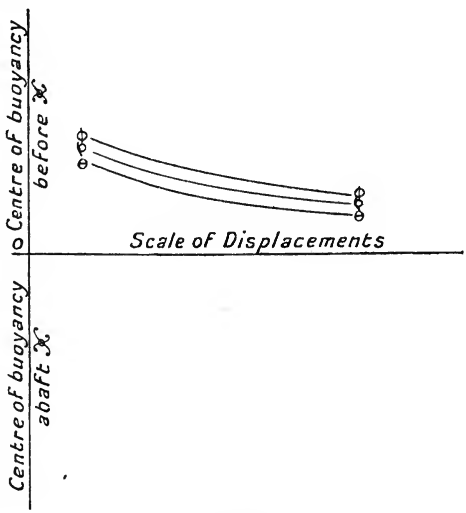


FIG. 59C.

If we take a series of water lines parallel to W_1L_1 we are able

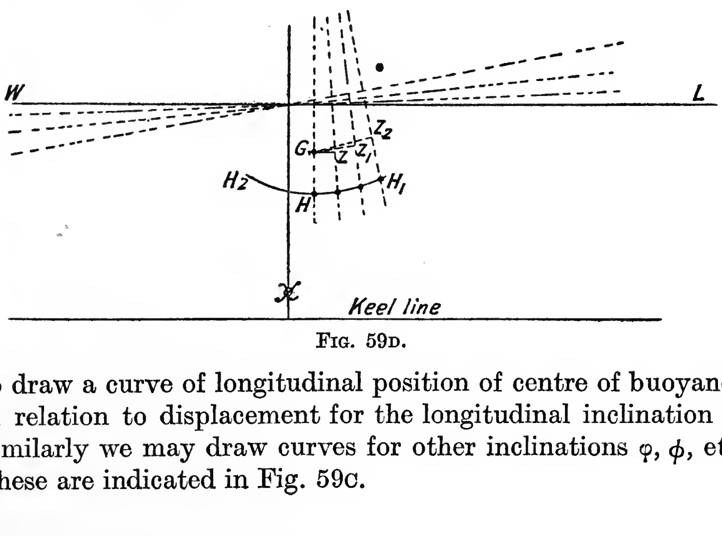


FIG. 59D.

to draw a curve of longitudinal position of centre of buoyancy in relation to displacement for the longitudinal inclination θ. Similarly we may draw curves for other inclinations φ, ϕ, etc. These are indicated in Fig. 59C.

In the same way we may show by means of curves the variation of the position of the centre of buoyancy above keel with relation to displacement and inclination; and, combining the two sets of curves, we are able to determine the absolute position of the centre of buoyancy for any particular displacement and longitudinal inclination, and for any specific displacement to draw on the profile the locus of centre of buoyancy when the angle of inclination changes. Such a locus is indicated in Fig. 59D by the curve H_2HH_1, where H represents the centre of buoyancy and G the centre of gravity for the level water line or normal condition of the vessel. If from several centres of buoyancy normals are drawn to the corresponding water lines, and from G perpendiculars Gz, Gz_1, Gz_2 be drawn to these normals, then Gz, Gz_1, etc., represent the righting arms for the corresponding inclinations of the vessel and denote the distances through which the centre of gravity must be moved to produce those inclinations.

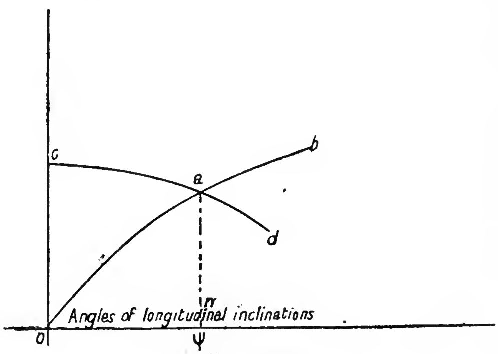


FIG. 59E.

Let *oab*, Fig. 59E, represent a curve of longitudinal $Gz \times W$, where W represents the actual displacement of the vessel after a weight w has been added. If w be moved parallel to the water line WL through a distance d, then $w.d \cos \theta$ is the inclining or tipping moment for an angle of inclination θ. Draw the curve *cad* of inclining moments, cutting the curve *oab* in a; the angle ψ represented by *an* in the figure is the longitudinal inclination of the vessel.

The late Mr. A. E. Long, M.A., introduced a very neat method* of "Trim Curves" by which to determine the trim of a vessel when the longitudinal inclination is not small. In calculating the displacements and positions of centres of buoyancy the principle of the "Bonjean" curves was employed.

In Fig. 59F let WL denote the level water line and H the

* Proceedings North East Coast Inst. of Engineers and Shipbuilders, 1906–7.

fore-and-aft position of the corresponding centre of buoyancy. Find the centres of buoyancy H_2 and H_4 corresponding to a trim (say) of 2 ft. and 4 ft. respectively by the stern for the same displacement. Set these points out on the level line WL in their correct relative position to H, and erect ordinates of 2ft. and 4 ft. as shown. Draw a curve T through their extremities and H—this is conveniently termed a "trim line"; from which it will be seen that if we know the fore-and-aft position of the centre of gravity relative to H and this be set out at G_1, then Y, the ordinate of the trim line at G_1, is the corresponding trim of the vessel under this condition.

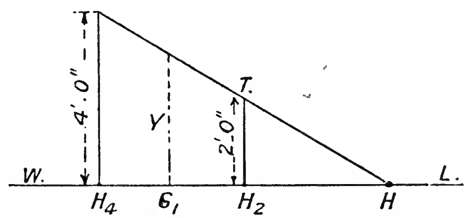


FIG. 59F.

To be exact, however, we must remember that the centres of gravity and buoyancy after the vessel has trimmed are not in the same line perpendicular to the keel or the level line WL, but in a line perpendicular to the new—inclined—water line,

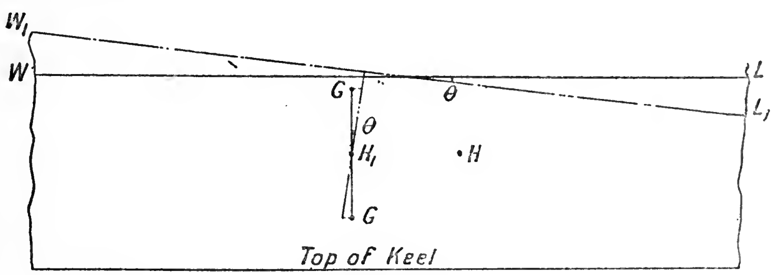


FIG. 59G.

and therefore a correction is required equal to $\pm$ GH_1 tan θ (Fig. 59G) in setting out the position of G_1, according as the centre of gravity (G) is above or below the centre of buoyancy.

With considerable longitudinal inclination there is also required a correction for mean draught, since the displacement curve is drawn to level lines and cannot now be used. This correction may be approximately obtained by taking the displacement to the inclined water line and comparing it with that to the level line ; the difference of displacement divided by the

area of water line will give the increase or decrease of the draught required for correction, in a manner similar to that described on p. 108.

We may now construct cross-curves of trim, as indicated in Fig. 59H, for a number of draughts ranging from above the load to below the light line. In the figure the lines $T_1, T_2, \ldots\ldots$ are the trim lines, corresponding to the level lines numbered 1, 2, etc., respectively, and the curves "$2a$ by the stern" and "a by the stern" are the cross-curves, relating to a trim of $2a$ and a feet respectively by the stern. If trims by the head

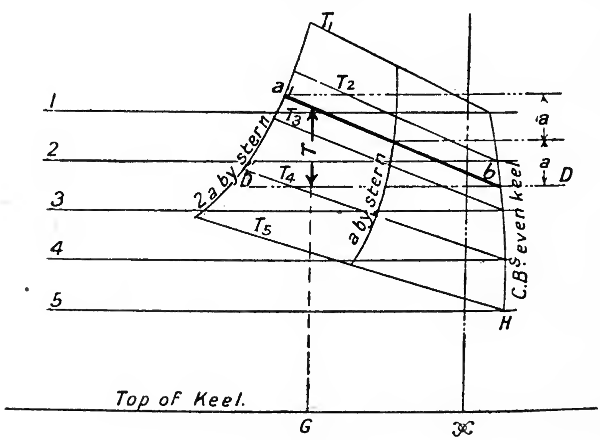


FIG. 59H.

are to be provided for, the trim lines may be continued to the right below their corresponding level lines.

We now have a diagram which covers the whole range of possible draughts and trims. *E.g.*, suppose we required to know the trim due to the draught D and the longitudinal position G of the centre of gravity. The trim line a_1b may be found by drawing level lines above the draught line DD at a distance of a and $2a$ to cut the cross-curves as shown; the required trim T is then found by drawing the vertical line through G to cut DD and ab.

For the purpose of approximate calculation it may be noted that the angle made by the tangent to the trim line at H to the level line HH_2H_4 (Fig. 59F) is about 42·5° (tan = ·916). Hence, if the centre of gravity is situated x ft. abaft the level centre of

buoyancy, the trim by the stern is $x \times \cdot 916$ ft.; or if a weight w be shifted d ft. and W = whole displacement, then the change in trim $= \dfrac{w \cdot d}{W} \times \cdot 916$

It might be that the lines of the vessel are not to hand just at the time when the calculation is required to be made. In such a case, if we have the metacentric diagram, we can by its aid usually determine with sufficient accuracy for practical purposes the change of trim resulting from putting a weight of considerable magnitude on board. We would proceed as follows :—

First determine the amount of sinkage on the assumption that the vessel will sink parallel to itself.

Let T be the tons per inch at the initial water line WL, then $\dfrac{w}{T}$ is the approximate increase of draught. Suppose W_2L_2 to be the new water line at which the tons per inch of immersion is T_2, then the mean tons per inch between the initial and new water line would be $\dfrac{T + T_2}{2}$ and the modified increase of draft $\dfrac{2w}{T + T_2}$.

In order for the above water lines to be parallel the added weight w must be placed in the same vertical line with the centre of volume of the added layer WW_2L_2L, and this centre of volume may be assumed to be situated midway between the centres of area of the water planes WL and W_2L_2.

In addition to an alteration—a rise—in the position of the centre of buoyancy of the ship the position of the centre of gravity also will be modified, rising or falling according as the weight w is placed above or below the level of the initial centre of gravity of the vessel.

The new longitudinal metacentric height—for the water plane W_2L_2—must be estimated (usually the new position of the centre of gravity may be disregarded and the metacentric height taken as the height of longitudinal metacentre above the centre of buoyancy) from which to obtain the moment to change trim one inch at the water line W_2L_2. Let this latter be denoted by M_2.

If the weight w be now shifted to its assigned position in the vessel through (say) the distance d, then the approximate change of trim is $\frac{w \cdot d}{M_2}$ and the new draughts may be determined.

Where a vessel is designed for a definite route on which, at least at certain ports, her draught must not exceed a certain amount, due perhaps to her having to pass over a bar, or through channels of limited depth of water, it will be necessary that her maximum load draught (generally aft) be less than an amount determined by the depth of water over the bar or in the channel.

For example, suppose a vessel loaded to the water line W_1L_1 is required to be so trimmed that the maximum draught (aft say) will be such that she may pass over a bar the depth of which below the water surface is x ft. Let WO (less than x) represent the required draught; draw WL parallel to W_1L_1, and suppose WL_2 represents the ultimate water line at which the vessel floats. Let w be the difference in displacement between the water lines W_1L_1 and WL, and T_1 and T denote the tons per inch corresponding respectively to these water lines, then $\frac{2w}{T + T_1}$ = the change of draught between W_1L_1 and WL $= LL_1$ (Fig. 59K). If g_1 denote the centre of volume of

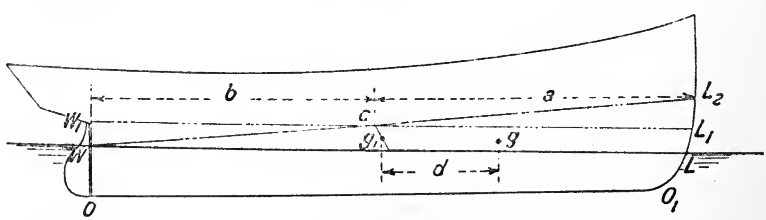


FIG. 59K.

the layer WW_1L_1L, and g the actual centre of gravity of that portion of the cargo or weights on board which has been shifted in order to bring about the desired change of trim, so that d is the fore-and-aft distance between g_1 and g, then, if M_1 be the moment to change trim one inch at W_1L_1, we have: Change of trim $= LL_2 = \frac{w \cdot d}{M_1}$. But if c be the centre of area of the water plane W_1L_1 distant a and b respectively from the bow and

stern, then : $L_2L_1 : L_2L = a : L$, where L is the length of the vessel, or $L_2L_1 = \frac{a}{L} \times \frac{w \cdot d}{M_1}$.

But $LL_1 = \frac{2w}{T + T_1}$.

$$\text{Therefore } L_2L_1 + LL_1 = \frac{a}{L} \times \frac{w \cdot d}{M_1} + \frac{2w}{T + T_1} = LL_2 = \frac{w \cdot d}{M_1},$$

$$\text{or } d = \frac{2\,L \cdot M_1}{(L - a)\,(T + T_1)} = \frac{2\,L \cdot M_1}{b\,(T + T_1)}.$$

If we now assume $T = T_1$ and $b = \frac{L}{2}$ as *approximately* correct, then $d = \frac{2M_1}{T}$ gives the distance through which the centre of gravity of the cargo (or weights) must be shifted to bring about the desired trim.

The movement of a weight already on board, or the placing of a weight on board, will affect the ship in a manner dependent upon the ultimate position occupied by the weight. The effect might be a change of trim only, transverse heeling only, or change of trim and transverse heeling combined, which may or may not be accompanied by an alteration of the position of the centre of gravity relative to the keel, *i.e.*, relative to a plane at or parallel to the keel and perpendicular to the middle line plane of the vessel.

Any modification of the position of the centre of gravity relative to the keel will affect the initial and statical stability, while the latter will also be affected by the movement of a weight transversely in such a manner that the position of the centre of gravity relative to the keel is preserved.

These points are extremely important, and have a direct practical bearing with regard to the disturbance or movement of mobile cargoes when a ship is rolling and pitching at sea. Generally the longitudinal movement of such cargoes is of minor importance, for the reason that they are usually confined between transverse bulkheads, spaced at intervals which are small compared with the vessel's length, and consequently their movement in the fore-and-aft direction must be limited, and the only effect would be a small change of trim accompanied by a slight modification of stability.

With displacement or movement of cargo in the transverse direction the case assumes greater importance, since there is likely to result inclination, which may be permanent or temporary, and consequently more or less dangerous, according to the nature of the cargo.

Too great care cannot be exercised in the disposal of such elements of freight, on account of their tendency to shift as the vessel rolls. Where large areas of free surface exist there is always an element of danger, and this applies both to liquids and such cargoes as grain, coal, etc. With the former there is an *immediate* change of surface as the vessel inclines, a portion

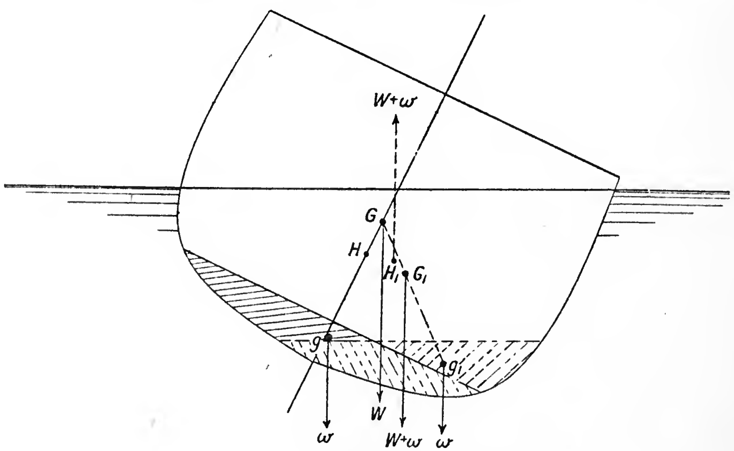


FIG. 60.

of the liquid moving in the direction towards which the vessel is inclining. With the latter the surface will only change when an angle of inclination is reached corresponding to the angle of repose of the mobile cargo, after which the particles will slide over each other, tending to heap themselves up on that side towards the direction of rolling.

It is in such cases as this that after passing through an oscillatory movement the vessel will come to rest at an inclination from the upright, and will remain in that position until perhaps, further oscillation being set up, the cargo may either re-adjust itself or move so as to cause a further inclination.

The consideration of the free surface is therefore one of great importance ; upon the dimensions, and especially the transverse

dimension, of that surface will depend the power of the cargo to cause the vessel to take up a position of rest inclined to the upright after the oscillation has subsided. In each case of change of surface with inclination the centre of gravity of the cargo changes, moving in the direction of angular displacement of the vessel, invariably diminishing or tending to diminish the righting arm.

Reducing the breadth of the free liquid surface or restricting the lateral movement of the mobile cargo is therefore tantamount to minimising the tendency of the vessel to upset.

This may be more clearly shown by considering Fig. 60, in which w is the weight of the contained fluid and g its centre of gravity ; W is the weight and G_1 the centre of gravity of the vessel. If the latter becomes inclined to an angle θ, so that the

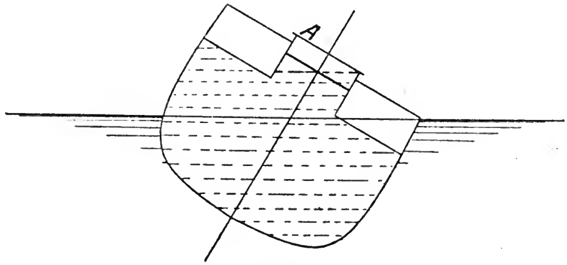


FIG. 61.

position of g be shifted to g_1, it will be seen that the movement of w might be such as to bring the common centre of gravity G_1 of the vessel and fluid to the right of the vertical line through the new centre of buoyancy H_1, thereby producing an upsetting moment.*

It is on this account that in double-bottom compartments utilised for carrying water ballast we find generally the lateral flow of any contained water restricted by longitudinals situated at the middle line and at positions intermediate to this point and the tank sides. The same principle holds in vessels designed for carrying oil in bulk ; an expansion trunk A surmounts the compartment (Fig. 61), serving the dual purpose of allowing the fluid to expand with increase of temperature and also limiting the freedom of lateral movement of the fluid.

* The formulæ for the stability of a vessel containing a mobile cargo has been fully discussed in Chapter I., p. 29.

With grain and other loose cargoes temporary wood bulkheads or shifting boards are built up in the holds of the vessel with a similar object.

A case very similar in principle to the foregoing is that of a vessel at sea shipping large quantities of water on to her deck; if efficient means are not provided by which to allow the water free and quick egress to the sea, the water will act as so much dead weight situated high up, raising the position of the centre of gravity, and at the same time lowering the "virtual" metacentre, both of which changes reduce the metacentric height.

A very important case of internal derangement of a vessel is where a compartment otherwise water-tight is laid open to the sea through bilging. In such a case the buoyancy of the

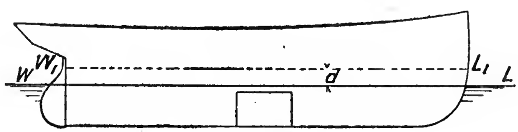


FIG. 61A.

compartment is lost. If, however, the latter has a water-tight crown or extend to a water-tight deck situated below the water plane, then, provided the deck does not suffer damage, that portion of the vessel's interior situated above the deck retains its buoyancy and will aid stability.

Again, if the compartment contain air which is shut in, so that the water which enters does not reach to the level of the water outside that portion of the air-space below, the exterior water surface retains its buoyancy, or if stores or cargo be contained within the compartment the amount of buoyancy lost will be V_1-v, where V_1 is the volume of the compartment (or the volume to the height of the initial water plane if the compartment extends above it), and v is the cubic content of the submerged cargo, less the volume of water which it absorbs.

Let A denote the mean area of water plane between the initial and final water lines, and a the mean horizontal sectional area of the compartment between the same water lines, or between the initial water line and the top of the compartment when the latter is situated between the initial and

final water lines. Suppose the compartment to be placed amidships. Three cases arise, viz. :—

(1) Where the initial water plane is situated above the crown of the compartment.

Let d denote the increase of draught then (Fig. 61A) :—

$$\text{Loss of buoyancy due to bilging} = V_1 - v.$$
$$\text{Apparent gain of displacement} = d \cdot A.$$
$$\text{Therefore } d \cdot A = V_1 - v$$
$$\text{or} \qquad d = \frac{V_1 - v}{A}.$$

(2) When the crown of the compartment is situated between the initial and final water planes (Fig. 61B).

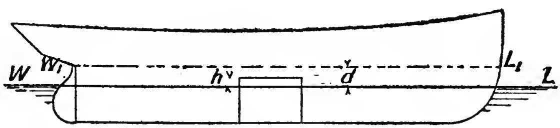


FIG. 61B.

$$\text{Loss of buoyancy due to bilging} = V_1 - v.$$
$$\text{Apparent gain of displacement} = d \cdot A - h \cdot a.$$
$$\text{Therefore } d \cdot A - h \cdot a = V_1 - v$$
$$\text{or} \qquad d = \frac{(V_1 - v) + h \cdot a}{A}.$$

(3) Where the crown of the compartment is situated above the final water plane (Fig. 61C).

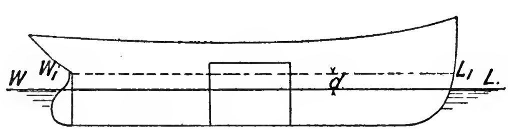


FIG. 61C.

$$\text{Loss of buoyancy due to bilging} = V_1 - v.$$
$$\text{Apparent gain of displacement} = d\,(A - a).$$
$$\text{Therefore } d\,(A - a) = V_1 - v$$
$$\text{or} \qquad d = \frac{V_1 - v}{A - a}.$$

For any other position of the compartment change of trim and, if the compartment does not extend right across the vessel, transverse inclination would result; in any case the stability of the vessel will be affected.

The volume of displacement remains unaltered, but, due to the deeper immersion of the vessel, the position of the centre of buoyancy relative to the keel will be raised, the centre of gravity of the vessel remaining unchanged.

Again, the moment of inertia of the water plane area in the case where it is broken by the compartment extending above it will be diminished in value, and therefore the height of the metacentre above the centre of buoyancy will be diminished. The difference between the *rise* of the centre of buoyancy and the *decrease* in value of the height of metacentre above centre of buoyancy will be the change in metacentric height (GM)—in other words, the metacentric height will be increased, remain the same, or be diminished, according as the increase in height of centre of buoyancy relative to keel is greater than, equal to, or less than the decrease in height of the metacentre above the centre of buoyancy.

In the case where the water plane area remains unbroken its moment of inertia will undergo no change—except that due to alteration of shape of water plane as the draught increases—and therefore the height of metacentre relative to the keel will increase just by the amount of the increase in height of centre of buoyancy above keel, which amount is also the increase in value of initial stability.

As the vessel is inclined from the upright the position of the centre of buoyancy will be modified as compared with that for the vessel with the compartment intact, and consequently the value of the righting arms will be affected.

It is interesting to note here the effect on the stability produced by closing the opening in the shell by which the water gained access to the compartment. There is now no communication between the water outside and that in the compartment: the contained water, therefore, may be regarded as so much weight added to that of the vessel while the buoyancy of the compartment has been restored, so that the vessel now floats with a volume of displacement increased just by the amount of the volume of the compartment less that of the

cargo $(V_1 - v)$, and the height of metacentre above centre of buoyancy will be altered by virtue of the change in volume of displacement and of the water plane regaining its lost area; while the position of the centre of gravity will be affected, due to the increased load.

The sum value of the effects due to these causes will give the initial stability of the vessel.

As the effect of damage to the skin of a vessel has such a special and practical bearing on the stability, the case of a bilged compartment of a rectangular vessel is worked out in detail below, the principles involved being equally applicable to vessels of ship-shape form.

Fig. 62 represents a rectangular or box-shaped vessel of length L and beam B, with a water-tight compartment *abcd*

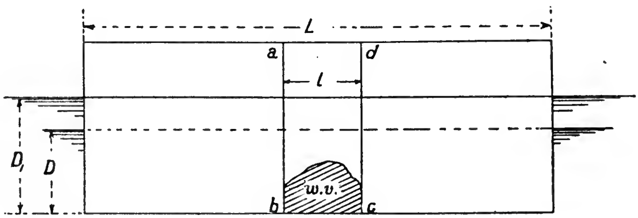


FIG. 62.

which is supposed to be damaged so that the water enters freely. The compartment is assumed to extend right across the vessel, to be situated at about amidships, and to contain cargo of volume v and weight w.

Let W denote the weight of vessel exclusive of w (that of cargo in the compartment).

Let D denote the mean draught of the vessel before damage.

Let D_1 denote the mean draught of the vessel after damage and when the vessel is again floating in equilibrium.

Let H, G, M, denote *generally* the centre of buoyancy, centre of gravity, and metacentre respectively.

(*a*) Before damage :—

Total weight or displacement of vessel $= W + w$.

Volume of displacement ,, $= L \times B \times D$.

$\therefore\ W + w = L \times B \times D \times w \ldots\ldots (1),$

where w = weight of water per cubic foot.

$$HM = \frac{\frac{1}{12} L \cdot B^3}{L \cdot B \cdot D} = \frac{B^2}{12D}$$

and height of metacentre above keel $= \dfrac{B^2}{12D} + \dfrac{D}{2}$.

(*b*) After damage :—

Total weight remains the same, viz., $W \times w$.

Volume of displacement, which is the same as before, is now represented by $(L - l)\, B \cdot D_1 + v$.

$\therefore\ W + w = (L - l)\, B \cdot D_1 \cdot w + v \cdot w \ldots\ldots (2).$

Equating (1) and (2) we get :—

$(L - l)\, B \cdot D_1 + v = L \times B \times D.$

$\therefore$ Change of mean draught $D_1 - D = \dfrac{l \cdot B \cdot D - v}{(L - l)\, B}$.

The centre of gravity remains unaltered.

The centre of buoyancy is changed, and its height above the keel is given by :—

$$\frac{D_1}{2} - \frac{\left(\frac{D_1}{2} - x\right) v}{(L - l)\, B \cdot D_1 + v} = \frac{D_1}{2} - \frac{\left(\frac{D_1}{2} - x\right) v}{L \cdot B \cdot D}$$

where x is the height of centre of gravity (and centre of buoyancy) of immersed cargo—considered homogeneous—above the keel.

$$HM = \frac{\frac{1}{12} (L - l)\, B^3}{(L - l)\, B \cdot D_1 + v} = \frac{(L - l)\, B^2}{12\, L \cdot D} = \frac{B^2}{12D_1};$$

and height of metacentre above keel is :—

$$\frac{(L - l)\, B^2}{12 \cdot L \cdot D} + \frac{D_1}{2} - \frac{\left(\frac{D_1}{2} - x\right) v}{L \cdot B \cdot D} = \frac{B^2}{12D_1} + \frac{D_1}{2} - \frac{\left(\frac{D_1}{2} - x\right) v}{(L - l)\, B \cdot D_1}.$$

(*c*) Suppose the damage made good or temporarily repaired, the enclosed water remaining as so much added weight.

Total weight $= W + w + (l \cdot B \cdot D_1 - v)\, w.$

Volume of displacement $= L \cdot B \cdot D_1.$

The centre of gravity will probably be altered in position, being raised or lowered relative to the keel according as the

centre of gravity of the enclosed water is situated above or below the original centre of gravity of the vessel.

The centre of buoyancy is now situated $\frac{D_1}{2}$ above keel, and there is a loss of moment of inertia of water plane area due to free water surface. We have :—

$$HM = \frac{\frac{1}{12} \cdot (L - l)\, B^3}{L \cdot B \cdot D.} = \frac{(L - l)\, B^2}{12\, L \cdot D_1};$$

height of metacentre above keel $= \frac{(L - l)\, B^2}{12 \cdot L \cdot D_1} + \frac{D_1}{2}.$

From the above particulars we can readily determine the loss or gain of initial stability. One particular case may be noted where the vessel is in light condition, *i.e.*, without cargo, in which case $x = o$ and $v = o$.

We obtain from :—

(*a*) Height of metacentre above keel $= \frac{B^2}{12D} + \frac{D}{2}.$

(*b*) ,, ,, ,, $= \frac{(L - l)\, B^2}{12\, L \cdot D} + \frac{D_1}{2}$

$$= \frac{B^2}{12D_1} + \frac{D_1}{2}.$$

(*c*) ,, ,, ,, $= \frac{(L - l)\, B^2}{12\, L \cdot D_1} + \frac{D_1}{2};$

and change of mean draught $= D_1 - D = \frac{l \cdot B \cdot D}{(L - l)\, B} = \frac{l \cdot D}{L - l}.$

To illustrate the foregoing principles we will consider a rectangular vessel of dimensions $100 \times 35 \times 10$ ft. floating at a mean light draught of 3 ft. in sea-water. Suppose that a compartment 5 ft. long stretching right across the vessel becomes bilged, and that the centre of gravity of the vessel is situated 4·5 ft. above the keel.

First.—Assume the compartment to be situated amidships.

(*a*) Before the vessel is damaged :—

The height of metacentre above keel is $\frac{35^2}{12 \times 3} + \frac{3}{2}$

$$= 35{\cdot}528 \text{ ft.}$$

Therefore the initial GM $= 35{\cdot}528 - 4{\cdot}5 = 31{\cdot}028$ ft.

(*b*) After the damage :—

The new draught is $D + \dfrac{l \cdot D}{L - l} = 3 + \dfrac{5 \times 3}{95} = 3{\cdot}16$ ft.

The height of metacentre above keel is $\dfrac{95 \times 35^2}{12 \times 100^3} + \dfrac{3{\cdot}16}{2}$

$= 33{\cdot}906$ ft.

Therefore the initial GM $= 33{\cdot}907 - 4{\cdot}5 = 29{\cdot}406$ ft.

(*c*) After the damage is repaired :—

The vertical position of the centre of gravity will be modified on account of the water now enclosed in the damaged compartment and is obtained as follows :—

Weight of vessel $= \dfrac{100 \times 35 \times 3}{35} = 300$ tons.

„ „ water in compartment $= \dfrac{5 \times 35 \times 3{\cdot}16}{35}$

$= 15{\cdot}8$ tons.

	Tons.	C.G. above keel.	Moment.
Weight of vessel	300	4′·5	1350·0 ft. tons.
Weight of water in compartment	15·8	1′·58	24·96 „
Total weight	315·8	4′·354	)1374·96 „

Height of metacentre above keel—

$$= \frac{(L - l)\, B^2}{12\, LD_1} + \frac{D_1}{2} = \frac{95 \times 35^2}{12 \times 100 \times 3{\cdot}16} + \frac{3{\cdot}16}{2} = 30{\cdot}69 \text{ ft.}$$

Therefore the initial GM $= 30{\cdot}69 - 4{\cdot}354 = 26{\cdot}336$ ft.

Second.—Suppose the compartment situated at the end of the vessel, say aft. In addition to a change of draught the vessel will trim by the stern.

(*b*) We will consider the vessel only after the damage is sustained.

The length of the vessel may now be taken as 95 ft., since the buoyancy of the end compartment is lost, and, first assuming the vessel to sink bodily without change of trim, the centre of buoyancy will move forward 2·5 ft., while the centre of gravity remains unchanged.

These points are represented by G and H_1 respectively in Fig. 62A, W_1L_1 representing the water line under these conditions.

If the vessel now assumes the position of equilibrium it will float at a water line W_2L_2, such that the ultimate position of the centre of buoyancy, H_2, will be situated vertically under G, and H_2G will be perpendicular to W_2L_2.

In the figure H_1m and mH_2 represent the movement of the centre of buoyancy perpendicular and parallel respectively

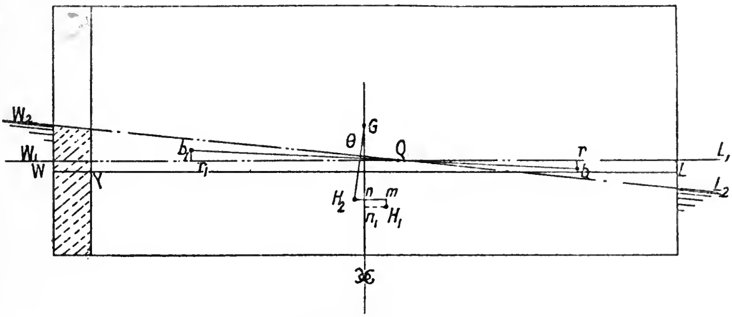


FIG. 62A.

to the keel, and the water lines W_1L_1, W_2L_2 will intersect at the point o, distant 2·5 ft. before the midship section.

H_1m will be small compared to H_2m and may be neglected. Let θ denote the angle of inclination of W_1L_1, to W_2L_2.

$$\text{Then } \tan\theta = \frac{H_2n}{Gn} = \frac{H_2n}{Gn_1} = \frac{H_2n}{4{\cdot}5 - 1{\cdot}58} = \frac{H_2n}{2{\cdot}92},$$

$$\text{or } H_2n = 2{\cdot}92 \tan\theta.$$

Now referring to Fig. 62A :—

$$\frac{H_2m}{r_1r} = \frac{\text{Volume of wedge of immersion (or emersion)}}{\text{Volume of displacement.}},$$

$$\text{or } \frac{H_2n + 2{\cdot}5}{95 \times \frac{2}{3}} = \frac{\frac{95}{2}\tan\theta \times \frac{95}{2} \times \frac{1}{2} \times 35}{95 \times 35 \times 3{\cdot}16}$$

$$\text{or } H_2n = \frac{95^3 \tan\theta}{3600} - 2{\cdot}5 = 2{\cdot}92 \tan\theta,$$

from which we obtain $\tan\theta = \dfrac{2{\cdot}5}{235{\cdot}24}$.

But change of trim forward

$$= \frac{95}{2}\tan\theta = \frac{95}{2} \times \frac{235{\cdot}24}{2{\cdot}5} = -{\cdot}505 \text{ ft.} = -6{\cdot}06 \text{ inches.}$$

and change of trim aft

$$= \left(100 - \frac{95}{2}\right) \tan\theta = \frac{52{\cdot}5 \times 2{\cdot}5}{235{\cdot}24} = +\ {\cdot}558 \text{ ft.} = +\ 6{\cdot}69 \text{ inches.}$$

i.e., draught forward $= 3{\cdot}16 - {\cdot}505 = 2{\cdot}655$ ft. $= 2$ ft. $7\frac{3}{4}$ in.

,, aft $= 3{\cdot}16 + {\cdot}558 = 3{\cdot}718$ ft. $= 3$ ft. $8\frac{5}{8}$ in.

The above solution has been obtained direct without employing the usual formulæ relating to calculations of trim; the latter method, however, is employed in the calculation given below.

The moment causing change of trim

$$= \text{weight of vessel} \times nm.$$
$$= 300 \times 2{\cdot}5 = 750 \text{ ft. tons.}$$

The moment required to cause one inch change of trim is given by

$$\frac{\text{Weight of vessel} \times \text{longitudinal metacentric height}}{12 \text{ (length on water line)}}.$$

Now longitudinal metacentric height

$$= \frac{\frac{1}{12} \times 35 \times 95^3}{95 \times 35 \times 3{\cdot}16} - 2{\cdot}92 = 235{\cdot}24 \text{ ft.},$$

where $2{\cdot}92$ is the vertical distance between G and H_2.

Therefore moment to change trim one inch is equal to

$$\frac{300 \times 235{\cdot}24}{12 \times 95} = 61{\cdot}9 \text{ tons.}$$

and the change of trim $= \dfrac{750}{61{\cdot}9} = 12{\cdot}12$ in.

or the change of trim forward $= -\,6{\cdot}06$ inches.

and ,, ,, ,, aft at Y $= +\ 6{\cdot}06$ inches.

and ,, ,, ,, aft at W $= \dfrac{\left(100 - \frac{95}{2}\right)}{\frac{95}{2}} \times 6{\cdot}06 = +\ 6{\cdot}69$ in.

These results agree, as we would naturally expect, with those obtained by the direct method given above.

An interesting case occurs where a water-tight compartment, such as a deep tank, becomes bilged so that the crown of the tank is situated at the water line at which the vessel ultimately floats.

Let D represent the draught corresponding to this water line.

As soon as the compartment is bilged the moment of inertia of the water plane is reduced from I to i, where

I refers to the *full* water plane area of the vessel, and

i „ „ *reduced* water plane area.

Suppose V to denote the ultimate volume of displacement, and suppose dV to denote a very small volume.

Then just before the draught D is reached the expression for

$$\text{HM} = \frac{i}{\text{V} - d\text{V}}.$$

If now from some cause (other than bilging), say, by placing a very small weight on board, the draught D be increased so that the volume of displacement is V + dv, the expression for

$$\text{HM}_1 = \frac{\text{I}}{\text{V} + d\text{V}},$$

since the position of the centre of buoyancy may be considered as unchanged, so that at, and in passing, the draught D, the height of metacentre above the centre of buoyancy, is altered by the amount.

$$\frac{\text{I} - i}{\text{V}}.$$

In designing a deep tank which, through bilging, may possibly be laid open to the sea, it will be advantageous to the vessel if the top of the tank is situated below the water line at which the vessel would float after the compartment has become bilged.

It is evident that the question of draught, trim, and stability resulting from one or more compartments being laid open to the sea through collision or otherwise is, in the case of a vessel either running in ballast or loaded, one of great moment; and if the vessel be of the passenger class it assumes very great importance indeed.

The safety of the ship and of the souls on board will depend upon the alteration in value of these characteristics, and if we add the probability of the existence of a rough sea at the time of the accident, the evil effects of a bilged compartment upon the seaworthiness of the vessel will be increased.

The principles which have just been discussed and applied to the rectangular vessel will require to be applied to the case of actual ships, the only difference being that with the latter

the problem refers to a body of more complex geometrical form.

Obviously we must so subdivide the vessel into water-tight compartments that safety may be reasonably presumed in the event of her meeting with accident at sea, and to do so it is necessary to lay down some definite standard or rule to guide us in assigning to the dividing partitions or bulkheads suitable positions in the ship. Practically speaking, we are not able, in determining upon such a standard, to embrace all the possible sources of danger to the flotability of a vessel, and all that can be done is to include the probable kinds of injury by the occurrence of which safety might be challenged, and in the design endeavour to minimise as far as may be their effect.

In 1891 the Board of Trade issued a Report as a result of the deliberations of a committee appointed by them to consider the question of the strength and spacing of bulkheads. This Report sought to set out by means of tables the spacing which should be given to water-tight bulkheads, so that, in the event of one compartment or two consecutive compartments—according to the grade in which the vessel under consideration lies—becoming bilged, the vessel would not sink in moderate weather below a certain line termed "the margin of safety line." This latter is defined as a line drawn round the sides of the vessel at a distance in the fore body of $\frac{3}{100}$ of the depth at side, and, merging at the after end to half that amount, *below* the bulkhead-deck line.

The bulkhead-deck is that water-tight deck to which the several bulkheads extend; it is assumed to be continuous, and that the surface of the water in the flooded compartment (which generally is level with the water outside) is not nearer to it than the margin of safety line.

The tables included in the Report give the maximum length of space which may be flooded, in terms of the distance of the centre of the space from the fore perpendicular, each measurement being reckoned in hundredths of the vessel's length.

The water line at which the vessel may float under normal conditions, determined by calculation from the Board of Trade freeboard tables, is dependent upon the moulded depth of the vessel, and the position of the bulkhead-deck (and therefore

the moulded depth or depth at side) is regulated by the conditions which the vessel has to fulfil (*e.g.*, it will depend upon the carrying capacity required). It is evident that freeboard will enter into the determination of the bulkhead spacing, and explains the marginal columns in the tables in which the minimum freeboard is given in terms of hundredths of the moulded depth. The tables were constructed for vessels carrying coal cargoes occupying 47 cubic ft. per ton, and on the assumption that 40 per cent. of the space in which the cargoes carried was free to admit water. Obviously, for cargoes of greater or less density than the above, the safety

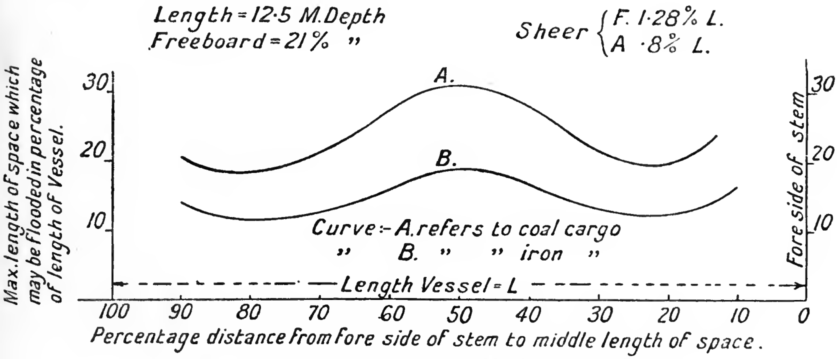


FIG. 62B.—Curves of Bulkhead Spacing for Sailing Ship.

of the vessel would suffer or be enhanced respectively, since the amount of water gaining access to the compartment would be greater or less than with a coal cargo.

To illustrate this point two curves have been selected from the Report, one relating to a coal cargo and the other to a cargo of pig iron carried by a sailing vessel ; these are shown in Fig. 62B.

The committee divided vessels generally into two classes, viz., high and low powered steamers, sailing vessels being also considered. The vessels were also separated into six grades according to their length, and the purpose for which they were designed ; they were considered to be capable of floating with two adjacent compartments, or one isolated compartment open to the sea, according to the grade in which the vessel was placed.

The proportionate length of space occupied by the machinery in high and low powered steamers was also set out as indicated in Fig. 62C and 62D.

The Tables I. and II. are abbreviated from the tables contained in the Report referred to above, and from Table II. the curve drawn in Fig. 62E has been deduced. This curve indicates the lengths of compartments which may be adopted

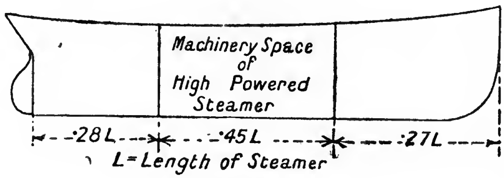


FIG. 62C.

in the case of a vessel of 12·5 depths to length, and a freeboard of 17 per cent. of the moulded depth. The profile, shown in Fig. 62F, illustrates without further explanation the manner in which the longitudinal sub-division of the vessel may be set out.

It may be added that with longitudinal bulkheads in combination with transverse bulkheads the committee considered that the vessel should not list so as to bring the deck on either

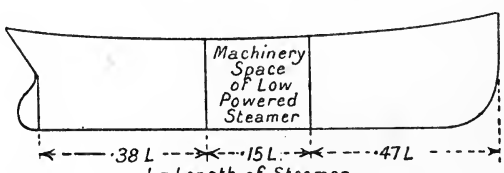


FIG. 62D.

side below the water level. It was further stated that every vessel coming under the above category should have her foremost bulkhead not less than $\frac{1}{20}$ length of the vessel from the stem measured on the load line.

The German Rules for the spacing of water-tight bulkheads in passenger steamers trading beyond the limits of Europe, published some eleven years after those of the British Board of Trade, are framed in a similar manner and need no comment here, except that no margin of safety line is given, but instead

Table I.,

Relating to High-powered Steamers.

L/D = 12·5. Sheer :—Forward = 1·28 per cent. L. Aft = ·8 per cent. L.

Coal Cargo.

Minimum freeboard in percentage of moulded depth.										Maximum length of space which may be flooded, in percentage of length of vessel.
21.	16·6	15·0	15·6	18·8	24·2	22·1	18·6	18·2	22·5	
23.	17·3	15·0	15·9	20·3	26·5	24·35	19·7	18·3	22·4	
25.	18·0	14·9	16·2	21·8	28·9	26·6	20·8	18·3	22·2	
27.	18·7	14·9	16·5	23·3	31·2	28·85	21·9	18·4	22·1	
29.	19·4	14·9	16·8	24·8	33·6	31·1	23·1	18·5	22·0	
	90.	81.	71.	61.	51.	41.	31.	21.	12.	

Percentage distance from fore side of stem to middle of length of space,* in terms of length of vessel.

Aft. Forward.

Table II.,

Relating to Low-powered Steamers.

L/D = 12·5. Sheer :—Forward = 1·28 per cent. L. Aft = ·8 per cent. L.

Coal Cargo.

Minimum freeboard in percentage of moulded depth.										Maximum length of space which may be flooded, in percentage of length of vessel.
17.	20·6	19·0	18·7	18·5	19·0	24·2	21·4	19·6	24·2	
19.	21·2	19·6	19·7	20·3	21·7	25·6	22·3	20·1	23·5	
21.	21·8	20·3	20·8	22·2	24·5	27·1	23·3	20·7	24·2	
23.	22·4	21·0	21·8	24·0	27·3	28·6	24·2	21·2	24·8	
25.	23·0	21·6	22·8	25·8	30·1	30·0	25·2	21·8	25·5	
27.	23·6	22·3	23·8	27·6	33·0	31·5	26·1	22·3	26·1	
29.	24·3	23·0	24·8	29·4	35·8	33·0	27·1	22·9	26·8	
	87.	81.	71.	61.	51.	41.	31.	21.	13.	

Percentage distance from fore side of stem to middle of length of space,* in terms of length of vessel.

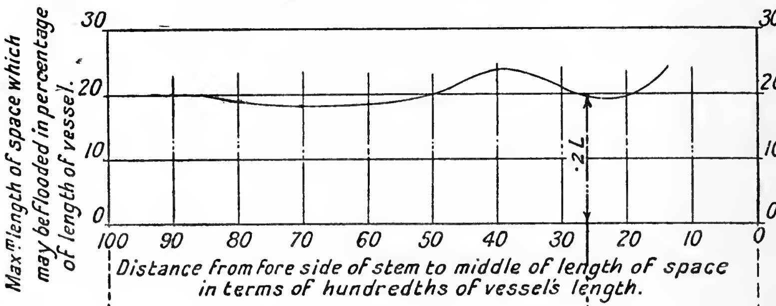
Aft. Forward.

* The space may consist of one or two compartments, according to the grade of vessel.

a certain percentage (depending upon the size of the vessel) of all spaces below the bulkhead deck is retained between that deck and the ultimate water line, this percentage being the minimum allowance for safety.

In arriving at their conclusions the committee assumed that

Fig. 62e.—Curve of Bulkhead Spacing for Low-powered Steamer corresponding to Table II. Freeboard = ·17 D.



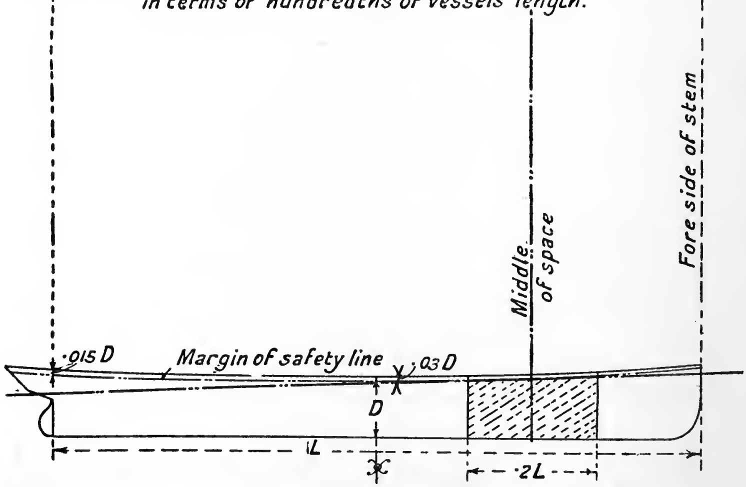


Fig. 62f.—Profile of Low-powered Vessel, showing Method of Setting out the Bulkheads, and effect of Flooding a Compartment.

the holds or cargo spaces were filled with a particular kind of coal, such that only a certain percentage of the cargo space admitted water. This assumption would probably hold good in the majority of cases of vessels, but at the same time would be inadmissible with passenger vessels and cargo vessels running light; in the event of these latter meeting with

accident by which compartments set aside as cargo spaces were opened to the sea, the *whole* contents of such compartments would be occupied by the inflowing water, to the level, generally, of the water outside.

In framing their recommendations some such assumption had to be made, and the tables were issued, apparently, not in any compulsory sense, but more as a guide to the shipowner and shipbuilder alike in determining the positions of bulkheads.

It has, however, been customary in the design of passenger vessels to make it a condition of the design that the committee's requirements referring to any two consecutive compartments be fulfilled, without consideration being given to the difference in condition existing between the actual vessel with much empty space in the holds and the standard vessel assumed to have her holds filled with cargo.

The two cases are distinct, and, obviously, the *spacing of bulkheads*, which, according to the committee's tables, would render a vessel still flotable after a compartment filled with cargo had become bilged, would require to be reduced in a passenger vessel of the same dimensions carrying little or no cargo in the holds.

With passenger vessels the positions of bulkheads should form the subject of separate and special calculation, based upon assumptions which embody, as far as practicable, all reasonable precautions against disaster consequent on accident to the hull of the vessel at sea.

It is generally necessary to fix upon the number and position of bulkheads in the early stages of the design as soon as the lines have been determined and the arrangement plans have been schemed out ; the general arrangement, indeed, will form a very important factor in determining the size of the compartments, and it becomes necessary, after the approximate positions of the partitions have been set out to satisfy the requirements of passenger accommodation, cargo space, bunkers, engine and boiler space, etc., to find out whether those positions will allow of any one or more compartments being flooded and the safety of the vessel be preserved.

To fix our ideas : suppose the vessel has been in collision and has received a blow at the bulkhead *a*, Fig. 62G, dividing the forward holds Nos. 2 and 3, so that these spaces are laid

open to the sea, it is required to know whether the ship will still float with the top of the bulkhead *c* above water.

The result is obtained tentatively.

The first approximation is made by assuming the holds (presumed initially empty) to have lost their buoyancy and estimating the position of the water line at which the vessel would float if sinking bodily in the water without change of trim. The corresponding centre of buoyancy is determined, and, knowing the moment to change trim one inch, we may calculate the new water line by determining the change of

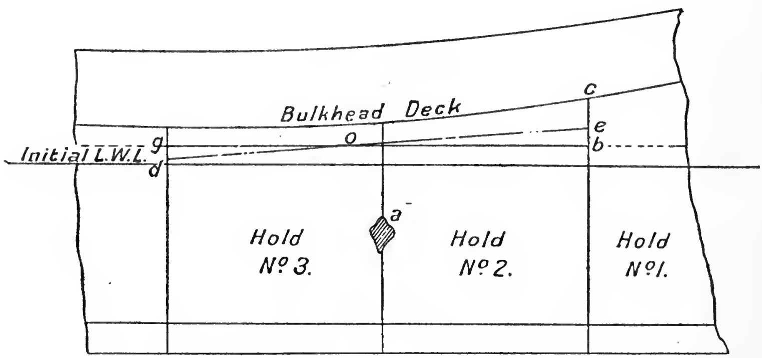


FIG. 62G.

trim. These water lines are indicated respectively by *gb* and *de*.

Generally the wedges *god* and *eob* are unequal, and in the present case the wedge *eob* is the greater, so that when the vessel trims, more water will enter the compartments than that indicated by the water line *gb*, the actual water line striking the bulkhead at the fore end of the compartments at a point *e* nearer to *c* than would be the case if the wedges were equal.

The position of the centre of buoyancy will also be modified by virtue of the trim by the head. A correction will therefore require to be made on these counts and a second approximation to the water line made, which, moreover, may be taken as substantially correct.

If we have originally fixed on a certain distance below the bulkhead-deck above which the water line must not rise, then where, in the process of calculation, this condition is not

fulfilled, the arrangement of the bulkheads must be modified accordingly. Such distance corresponds with the "margin of safety line" referred to on p. 130, and would be determined by a prearranged minimum amount of reserve buoyancy and transverse stability necessary to ensure the vessel remaining afloat under normal weather conditions.

The stability must now be examined, bearing in mind that, since the compartments are laid open to the sea, the moment of inertia of the water plane area enclosed within the compartments is lost.

The stability will be affected in the following manner :—

(*a*) The position of the centre of buoyancy will be altered by virtue of the deeper immersion and trim of the vessel, and the loss of buoyancy of the bilged compartments.

(*b*) If the bilged compartments contain cargo, the position of the centre of gravity will be changed, due to the loss of weight in amount equal to that of the water displaced by the cargo.

(*c*) The displacement is diminished by the amount stated in (*b*).

(*d*) Loss of moment of inertia of water plane area.

A case of practical importance in which the stability of a vessel may so change as to jeopardise her safety is that in which she is partially or wholly supported by the blocks in docking on account of the water being pumped out so quickly as to allow of no (or insufficient) support by means of shores; or in the case of a vessel taking the ground on a falling tide. It requires little explanation to show in what way loss of stability is brought about.

A vessel so situated instead of being wholly supported by the buoyancy of the water is now supported by the docking blocks or the ground and the water acting in conjunction, the sum of the resulting upward forces—viz., the reaction of the ground, say—and the buoyancy of the water, being equal to the weight of the vessel. It is obvious that there is a loss of buoyancy just equal to the upward pressure of the ground as compared with the buoyancy when the vessel is floating freely; further, it means that a force equal in value to the lost buoyancy, instead of acting at a point near the water

plane, and so ordinarily having a righting moment, is now acting at the keel or at the point or line of support near it with a tendency to upset the vessel. This will be better understood by reference to Fig. 63.

Let WL be the water plane when the vessel is floating upright, and suppose after taking the ground the water plane falls to W_1L_1. As a result the vessel will heel over under the action of external forces, so that w_1l_1 is now the water plane

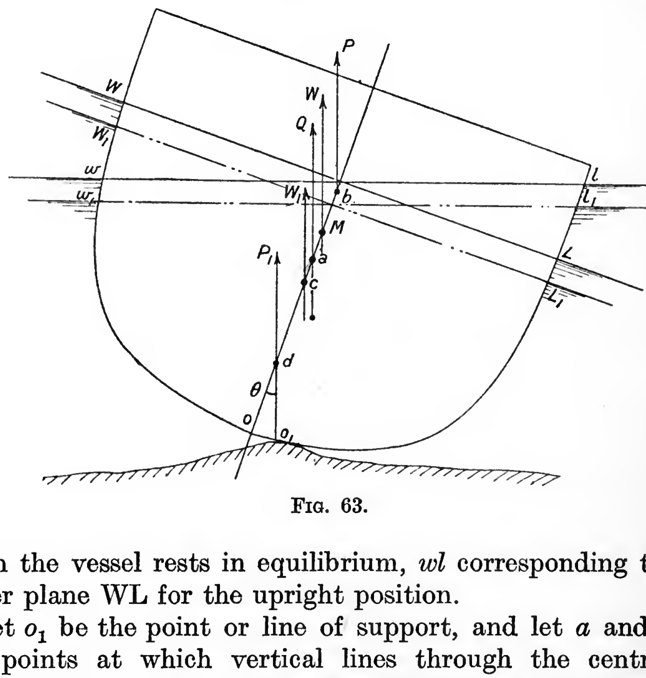


FIG. 63.

when the vessel rests in equilibrium, wl corresponding to the water plane WL for the upright position.

Let o_1 be the point or line of support, and let a and b be the points at which vertical lines through the centres of gravity of the *actual* displaced water w_1ol_1 and the slice or zone ww_1l_1l respectively intersect the middle line plane oab. Let Q and P denote the forces (due to buoyancy) which would act upwards through these points if the vessel were floating freely at the angle of inclination θ. Then :—

$$Q + P = W,$$

whereof W (the buoyancy of the vessel) would act upwards through the metacentre M, corresponding to the water line WL.

However, P acting at b is non-existent, since the buoyancy of the zone ww_1l_1l is lost—and is replaced by P_1 (= P) acting

vertically upwards at o_1; consequently the resultant of Q acting at a and P_1 acting through d is a force W_1 (= W) acting through some point c in oab, resulting in a loss of metacentric height "cM," and in a loss of statical stability $W \times c\text{M} \sin \theta$. For a small inclination, o_1 would practically coincide with o, and therefore d and o may be taken as coincident.

$$\text{Therefore } ca = ad \cdot \frac{P_1}{W} = ao \cdot \frac{P}{W},$$

$$\text{and } a\text{M} = ab \cdot \frac{P}{W}.$$

$\therefore$ Loss of metacentric height

$$= c\text{M} = ca + a\text{M} = \frac{P}{W}(ao + ab) = \frac{P}{W} \cdot d,$$

where d is the height above keel of the centre of buoyancy of the *lost buoyant portion* ww_1l_1l.

The loss of statical stability

$$= W \times \frac{P}{W} \cdot d \times \sin \theta = P \cdot d \sin \theta.$$

For any large angle of heel it is obvious that—at least in vessels with flat plate keels—there will be an appreciable distance between o and o_1, and that d cannot then be considered as coinciding with o. The loss of statical stability under such conditions must be considered in a manner similar to that described on p. 35. However, in all cases one condition of equilibrium is that the moment of the weight of vessel is equal to the moment of the *actual* buoyancy of the vessel about the point or line of support.

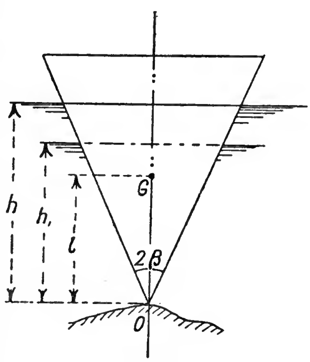


Fig. 64.

If the vessel cannot be considered as being supported more or less along the line of keel, but at a point towards either end, as in the case of docking a ship trimming by the stern, or where a vessel runs ashore so as to be supported at the bow, then a change of trim, in addition to loss of statical stability, will result.

As illustrating this part of the subject we will take the

case of a vessel of triangular section apex downwards, resting on rough ground. Find the depth of water over the ground in order that the vessel may be in stable equilibrium, and find the condition for neutral equilibrium.

Let L = length of vessel.
,, h = draught of vessel when floating freely.
,, h_1 = ,, ,, ,, under constraint.
,, l = height of centre of gravity of vessel above line of support o.

We have :—

Displacement of vessel when floating freely—

$$= \mathrm{W} = w \times \mathrm{L} \cdot h^2 \tan \beta.$$

Displacement of vessel when floating under constraint—

$$= \mathrm{Q} = w \times \mathrm{L} \cdot h_1^2 \tan \beta.$$

New position of centre of buoyancy under constraint—

$$= \frac{2}{3} h_1 \text{ above } o.$$

Height of metacentre above centre of buoyancy for the aground or constrained condition

$$= \frac{\frac{1}{12} \cdot \mathrm{L} \cdot (2h_1 \tan \beta)^3}{\mathrm{L} \cdot h_1^2 \tan \beta} = \frac{2}{3} h_1 \tan {}^2\mathrm{B}.$$

$$i.e., \text{ equal to } \frac{2}{3} h_1 + \frac{2}{3} h_1 \tan {}^2\beta = \frac{2}{3} h_1 \sec {}^2\beta \text{ above } o.$$

∴ For stability we must have :—

$$\mathrm{Q} \times \frac{2}{3} h_1 \sec {}^2\beta > \mathrm{W} \times l$$

$$\text{or } w \times \mathrm{L} \cdot h_1^2 \tan \beta \times \frac{2}{3} h_1 \sec {}^2\beta > w \times \mathrm{L} \cdot h^2 \tan \beta \times l.$$

$$i.e., h_1 > \sqrt[3]{\frac{3h^2 l}{2 \sec {}^2\beta}}.$$

and for neutral equilibrium :—

$$h_1 = \sqrt[3]{\frac{3h^2 l}{2 \sec^2 \beta}}.$$

An interesting example is that of a vessel of parabolic section resting on a rough rock bottom ; it is required to find the depth of water over the rock in order that the vessel may be in stable equilibrium, having given l, the height of centre

of gravity of the vessel above the keel, h_1, the draught when floating freely, and L, the length of the vessel.

Let $y^2 = 4\,ax$ be the equation to this parabolic section boundary, then differentiating we have :—

$$\frac{dy}{dx} = \frac{2a}{y} \text{ and } \frac{d^2y}{dx^2} = -\frac{4a^2}{y^3}.$$

therefore the radius of curvature at a (the keel) is given by :—

$$\mathrm{R} = \frac{\left\{1 + \left|\frac{dy}{dx}\right|^2\right\}^{\frac{3}{2}}}{\frac{d^2y}{dx^2}} = \frac{\left(1 + \frac{4a^2}{y^2}\right)^{\frac{3}{2}}}{-\frac{4a^2}{y^3}} = 2a, \text{ since } y = 0.$$

If C is the centre of curvature at a, $\mathrm{C}a = 2a$, and for a small

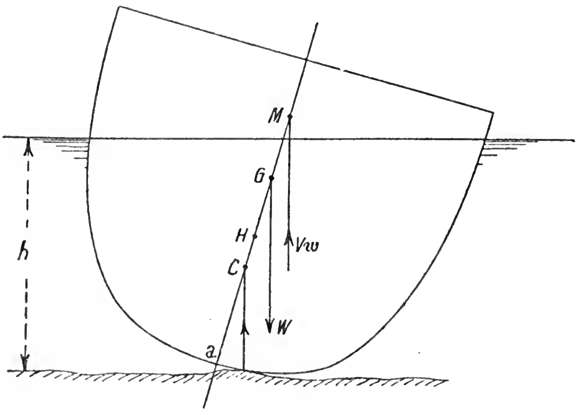


FIG. 65.

inclination the reaction of the ground passes through C. Let H,G, and M denote the centre of buoyancy, centre of gravity, and metacentre respectively ; H and M referring to the draught h or depth of water over the rock required.

Now $a\mathrm{H} = \frac{2}{3}h$.

The moment of inertia of waterplane area

$$= \frac{1}{12} \cdot \mathrm{L} \cdot 4^3\,(ah)^{\frac{3}{2}} = \frac{16\mathrm{L}}{3}\,(ah)^{\frac{3}{2}}.$$

and volume of displacement $= \frac{8}{3} \cdot \mathrm{L} \cdot a^{\frac{1}{2}}\,h^{\frac{3}{2}} = \mathrm{V}.$

Therefore $$\text{HM} = \frac{\frac{16\text{L}}{3}(ah)^{\frac{3}{2}}}{\frac{8\text{L}}{3}\cdot a^{\frac{1}{2}}\,h^{\frac{3}{2}}} = 2a,$$

and $\text{CM} = a\text{H} + \text{HM} - aC = \frac{2}{3}h + 2a - 2a = \frac{2h}{3}$,

and $\text{CG} = l - 2a$.

Taking moments about C we have the condition for stability:

$$\text{V}\cdot w \times \frac{2h}{3} > \text{W}\ (l - 2a).$$

Now $$\text{W} = \frac{8}{3}\cdot\text{L}\cdot a^{\frac{1}{2}}\,h_1^{\frac{3}{2}}\,w.$$

$$\therefore\quad \frac{8\text{L}\,a^{\frac{1}{2}}}{3}\cdot h^{\frac{3}{2}} \times \frac{2h}{3} \times w > \frac{8\text{L}\,a^{\frac{1}{2}}}{3}\cdot h_1^{\frac{3}{2}} \times w\ (l - 2a),$$

or $$h^5 > \frac{9\ (l - 2a)^2}{4}\,.\,h_1^3.$$

If we take the case where the form of the section is given by the equation $y^2 = 20x$, and $l = 18$ ft. and $h_1 = 20$ ft.

We have as the condition for stability on grounding that :—

$$h^5 > \frac{9}{4} \times 8^2 \times 20^3,$$

or $$h > 16{\cdot}3 \text{ ft.}$$

that is, the depth of water over the rock must be greater than 16·3 ft.

A case of constraint met with in practice, and closely resembling the case of a vessel aground or having partially taken the blocks in docking, is that of a vessel during the process of launching. Until that stage is reached at which she becomes wholly water-borne she is supported partly by the water and partly by the ways and poppets.

If through any extraneous cause during launching there should be a tendency of the vessel to heel or rock, this generally would be amply resisted by the action of the ways, and danger of heeling is only likely to occur at or just previous to the vessel leaving the ways when the supporting forces other than that of the water are reduced to a minimum. It has been seen that the vessel under such a condition possesses less stability than when entirely buoyant, and it is necessary to ensure that she

possesses sufficient stability for the buoyant condition in order to obviate any danger to overturn on account of this reduction.

It is for this reason that a preliminary calculation of stability is sometimes made for the launching condition. It is not always done, however, as the shipbuilder has generally sufficient data and experience of previous vessels of similar type to guide him ; but in the case of special types, where perhaps there is a possible doubt as to safety, such a calculation should be made.

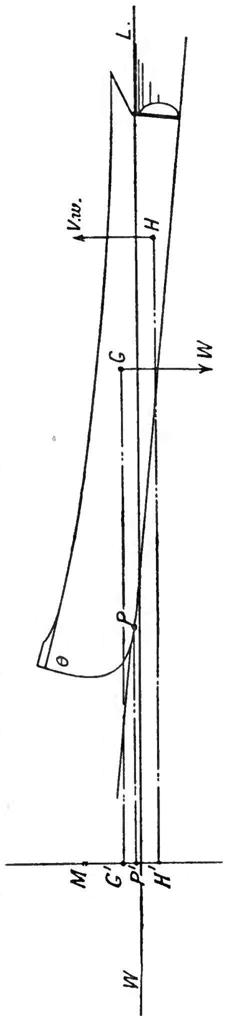


FIG. 65A.

At the time of launching the weight of a vessel is generally considerably less than that of the designed light ship ; usually she has little machinery on board, and the internal arrangement and fittings are far from being completed ; consequently the centre of gravity is situated much higher than will ultimately be the case in the finished vessel.

The small displacement of the vessel will be accompanied by a comparatively small moment of inertia of water plane area, the relation between the two generally being such as to give a value for the height of metacentre above centre of buoyancy greater than under any condition hereafter. The centre of buoyancy, however, is situated very low, and consequently the metacentre may be so situated relative to the centre of gravity as to result in a small metacentric height, which, in order to ensure safety, might necessitate the addition of weight or ballast placed low down in the vessel prior to launching.

In such a case the effect of the added weight upon the draught and trim of the ship when floating freely would also require to be carefully calculated.

As the vessel proceeds down the ways the displacement,

trim, and stability will be continuously changing, until she becomes entirely water-borne at the moment she becomes clear of the ways.

The probability is that in most cases a vessel at the time of launching possesses a considerable amount of metacentric height and that ballast is seldom required.

Let Fig. 65A represent a vessel at such a position in her movement down the ways that she is partially water-borne and on the point of hinging or pivoting about an axis at P situated at the forward poppets.

Let G denote the centre of gravity, and H the centre of buoyancy of the immersed portion of the vessel, WL being the water surface.

Let $H,^1 P,^1$ and G^1 be the projections of H,P, and G respectively on a vertical transverse plane; and let M denote the position of the metacentre relative to H^1, such that $H^1M = \frac{i}{v}$,

where i denotes the moment of inertia of the water plane area.
v ,, ,, volume of water displaced.

If the vessel tends to incline transversely it will be about an axis through P, and for a small inclination $d\theta$ the righting moment would be :—

$$(v \,.\, w \,.\, MP^1 - W \,.\, G^1P^1)\, d\theta = W \,.\, d\theta \left(\frac{v \,.\, w}{W} \,.\, MP^1 - GP^1\right),$$

the factor inside the brackets corresponding to the metacentric height of the vessel when afloat.

The causes affecting the stability of a vessel at the time of launching are: (1) Disturbed water surface; (2) unequal pull on the drags; (3) movement of weights on board; (4) buoyancy of the sliding ways.

In the Report made by Sir E. J. Reed on the "*Daphne*" launching disaster it was stated that at the time of launching the estimated metacentric height was about *four* inches, a margin too small to set against the possibility of inclination of the vessel as she left the ways, notwithstanding the extent of the exposed sides or the great amount of freeboard at the time; indeed, vessels under such conditions* do not necessarily possess large values of righting moment at considerable angles of inclination, nor large ranges of stability.

* Small metacentric height and large amount of freeboard when light.

It is better, therefore, to construct statical stability curves rather than to take only the value of initial stability as the

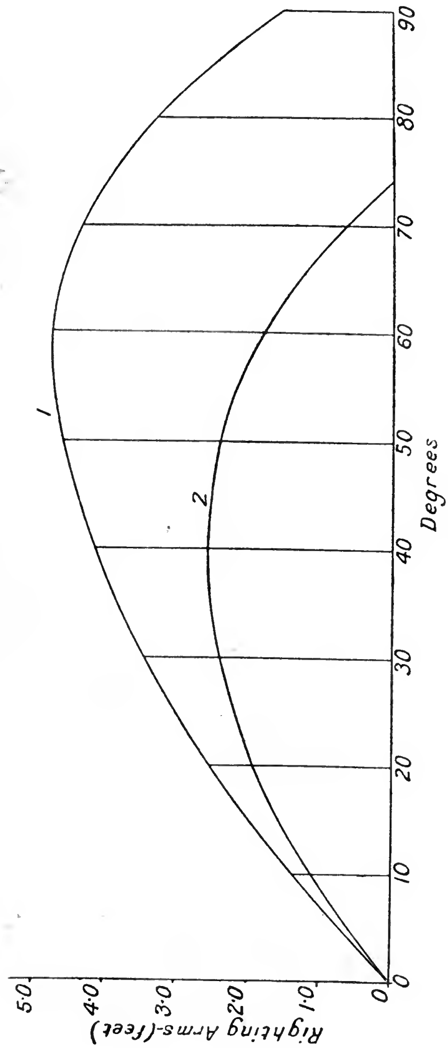


FIG. 66.

Curve 1, relating to T.S. first-class passenger vessel. Launching weight, 5,650 tons; mean draught, 12 ft. 6 in.; freeboard 23 ft.; ballast on board 200 tons; calculated C.G. above base 21·7 ft., initial G.M. 7·57 ft.

Curve 2, relating to T.S. first-class passenger vessel. Launching weight, 1,640 tons; mean draught 9 ft.; freeboard 12 ft. 2 in.; ballast on board, 160 tons; calculated C.G. above base 15·84 ft., initial G.M. 5·76 ft.

standard by which to judge of the righting power of the vessel in the event of her becoming inclined.

Fig. 66 shows the stability curves got out for two passenger vessels at the time of launching. In each case there is a

considerable amount of initial stability, accompanied by a good value of righting arm and range of stability.

Curve 1 may be compared with Fig. 42, relating to the same vessel under various conditions of loading, but with much less value of initial stability.

Before passing on to the consideration of dynamical stability mention might be made of the effect of the propeller and of the rudder in modifying the stability of a vessel when under steam ; the points are generally of interest, but in the case of a small swift vessel of moderate metacentric height a not inconsiderable influence on the stability might be exercised.

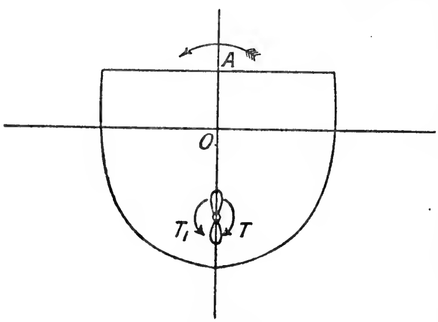


FIG. 67.

Consider the case of a single - screw steamer, where T represents the turning couple of the propellers ; the fluid resistance opposing the circular motion of the propellers sets up a couple T_1 of opposite sign. The effect is to rotate, or to tend to rotate, the vessel in the direction of T_1, or, as shown at A, about some longitudinal axis O.

Suppose that as a result the vessel is inclined through an angle of steady heel θ, thereby setting up a righting couple $W \times GM \sin \theta$.

$$\text{Then } T = W \times GM \sin \theta.$$

If H be the effective horse-power of the engines the amount of work done per minute is :—

$$\frac{33{,}000 \text{ H}}{2{,}240} = 2 \pi n \text{ T} = 14{\cdot}73 \text{ H ft.-tons}$$

or $$\text{T} = \frac{14{\cdot}73 \text{ H}}{2 \pi n}.$$

where n = number of revolutions per minute,

So that $$W \times GM \sin \theta = \frac{14{\cdot}73 \text{ H}}{2 \pi n}$$

or $$\sin \theta = \frac{14{\cdot}73 \text{ H}}{2 \pi n \,.\, W \,.\, GM}.$$

With a twin or quadruple screw vessel, where the corre-

sponding propellers turn in opposite directions, no heeling effect would result. With paddle steamers there is a similar tendency to incline, but in the longitudinal direction, the bow tending to rise or to become more deeply immersed according as the vessel is going ahead or astern respectively.

As an example we will take the case of a single-screw steam yacht of 560 tons displacement. GM $= 2{\cdot}3$ ft.

Effective horse-power $650 = H$.

Revolutions per minute $135 = n$.

we have
$$\sin\theta = \frac{14{\cdot}73 \times 650 \times 7}{2 \times 22 \times 135 \times 560 \times 2{\cdot}3}.$$
$$= {\cdot}00785.$$
or
$$\theta = 27' = \text{about half a degree.}$$

The angle of inclination for a large single-screw passenger vessel of 14,500 tons displacement, 8,500 horse-power, 85 revolutions per minute, and GM $= 2{\cdot}0$ ft. would be nearly half a degree; whereas in the case of swift vessels of comparatively small displacement with large values of horse-power and revolutions, the inclination may approach two to three degrees.

When a vessel is steaming at sea the presence of waves may have an unfavourable effect on its stability. With certain small swift vessels when run at full speed a change in stability accompanied with tenderness has been noticed. In such cases the nature of the water surface is completely altered, generally producing a change in the area and moment of inertia of the area of the water surface section with the vessel. In conjunction with this we must consider the effect of the wave motion in augmenting the hydrostatic pressure at those sections where the immersion is reduced and in diminishing the pressure where the immersion is increased, virtually altering the density of the water and probably resulting in a modification of the position of the centre of buoyancy. It is evident that there may or may not accrue a result prejudicial to the stability of the vessel when the effect of these several factors are combined.

With reference to this point Sir J. I. Thornycroft, F.R.S., in a paper on "Torpedo-boat Destroyers" read before the Institution of Civil Engineers in 1895, states :—

"There is a noticeable change in the conditions of stability

at full speed. The vessel appears to be more tender; and although this impression is perhaps chiefly due to the greater heeling effect of small movements of the helm at high speed, the stability is *actually* reduced as compared with the normal still-water condition.

"The change in the water line, which falls in wide part of the vessel at amidships and rises at the finer parts of the bow and stern, lowers the metacentre.

"Vessels of small initial stability and of a form which when driven at high speed causes the water to pile up at the bow and stern and fall considerably amidships have been known to become unstable when so driven."

Sir J. Thornycroft goes on to say that the change in metacentric height due to the wave profile in figure 8 of the paper referred to, and which is reproduced here in Fig. 67A, is two

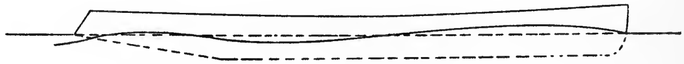


FIG. 67A.

inches, assuming that the hydrostatic pressure at each immersed section remains unchanged by the wave motion.

The effect in the case of large vessels would probably not be very apparent, due to their large displacement and the fact that the area of water surface section of the vessel is more or less preserved.

Reference to this point is made by Sir W. H. White in his "Manual of Naval Architecture," in which he states that "as a rule it has not been found necessary to calculate curves of stability for these conditions," and further refers to his joint work with the late Mr. John at the time of the loss of the *Captain*, and to some investigations made by M. Ferand in the case of French torpedo-boats.

Putting the rudder over when the vessel is in motion brings into being forces which may result in heeling effect. In such a case there is transition from straight line to curvilinear motion, accompanied by the creation of centrifugal force acting outwards through the centre of mass of the vessel, and which, in combination with the pressure on the rudder and that on

the surface of the vessel while turning, may or may not heel the vessel, according to the relative values, directions, and points of application of these forces. Further reference will be made to this point in the following chapter dealing with submarines.

Reference will now be made to "Dynamical Stability," or the amount of work performed in heeling a vessel over to a given angle from the initial position of rest.

In the case of a body acted upon by a force in a certain definite direction so that movement of the body results, the work done is represented by the product of the force and the distance through which the body has moved in the direction of the force, the movement of the body being referred to that of its centre of gravity.

When we come to consider the angular movement of a ship, the forces collectively contributing to the restoration of equilibrium when the vessel is inclined are vertical—one the weight of the vessel, acting vertically downwards through the centre of gravity; the other, due to buoyancy and equal in value to the weight of the vessel, acting vertically upwards through the centre of buoyancy, whereof the centre of gravity is a fixed point in the ship (but is variable in space according to the inclination), while the centre of buoyancy is a point variable in position both relative to the vessel and in space.

If during a given angular movement we could determine the vertical distances moved through separately by these points, reckoning the distance *positive* in the direction of the force and *negative* in the opposite direction, then the algebraic sum of such distances multiplied by the weight or displacement of the vessel would be the amount of work done on the vessel by the external forces causing inclination, or *vice versâ*—if the external forces are removed, would represent the amount of work done by the weight and buoyancy of the vessel in restoring her to the normal position of equilibrium.

This will be better understood by reference to Fig. 68, in which WL and W_1L_1 respectively represent the water lines for the normal and inclined positions, G the centre of gravity, H the centre of buoyancy for the normal, and H_1 for the inclined position (θ).

It is evident that (ZH_1 — HG) represents the distance by

which the two points, viz., the centre of gravity and centre of buoyancy, have separated vertically during the inclination θ, and $W(ZH_1 - HG)$ the amount of work done on the vessel.

We can find an expression for $(ZH_1 - HG)$. If w represents the displacement in tons of either of the wedges WSW_1, LSL_1,

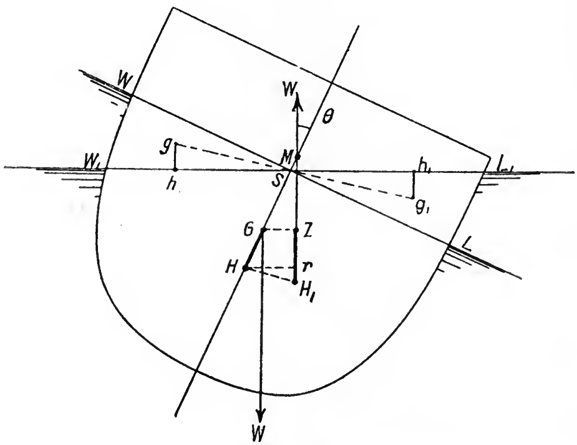


FIG. 68.

g and g_1 their respective centres of gravity, then if gh, g_1h_1 be drawn perpendicular to W_1L_1 we have :—

Fall of centre of buoyancy relative to W_1L_1 (the water surface)

$$= \frac{w}{W}(gh + g_1h_1) = rH_1,$$

and $Zr = GH \cos \theta$.

$$\therefore \quad ZH_1 - HG = HG \cos \theta + \frac{w}{W}(gh + g_1h_1) - HG,$$

and the work done $= w(gh - g_1h_1) - W.HG\,(1 - \cos \theta)$— Moseley's formula for dynamical stability.

The difficulty with this formula is that the term $w(gh + g_1h_1)$ would have to be calculated for each specified angle of heel and for each displacement for which the dynamical stability is required, and to obviate this the dynamical stability is obtained in another way—from the consideration of the righting arm or moment.

It may easily be proved by the principles of dynamics that if a couple act on a body capable of rotation about an axis the

direction of which is perpendicular to the plane of the couple, and a small angular movement results, the work done on the body is given by $K \cdot d\theta$, where $d\theta$ denotes the indefinitely small angle turned through by the body, K denotes the value of the couple, which during the movement may be considered constant. K, however, may vary from moment to moment, and if the whole movement be denoted by ϕ then the total amount of work done will be :—

$$\int_o^\phi K \,.\, d\theta,$$

or if we desired the amount of work performed from the time

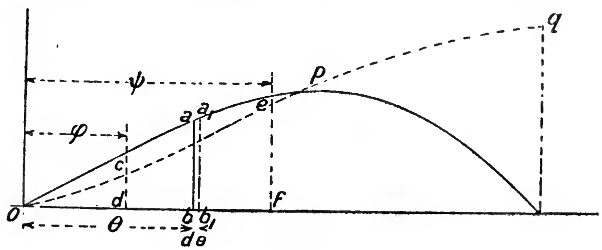


Fig. 69.

when the angular movement was φ to that when it had reached ϕ, the expression would be

$$\int_\varphi^\phi K \,.\, d\theta.$$

Applying this to the case of a vessel inclined from her normal position of rest, K would represent the righting couple $W \times GZ$—a variable quantity, GZ being the righting arm at the particular angle of inclination θ. $W \cdot GZ \cdot d\theta$ would represent the work done during the small increase or decrease $d\theta$ of the angle of inclination, and $\int_\varphi^\phi W \cdot G\,Z \cdot d\theta$ the work done between the inclinations φ and θ.

Let Fig. 69 represent the statical stability curve for the vessel plotted to ordinate values of righting arm; then, if ab, a_1b_1 represent two ordinates corresponding to the inclinations θ and $\theta + d\theta$, the area abb_1a_1 is $ab \times d\theta$, and the area of the whole diagram enclosed between the ordinates cd and ef is $\int_\varphi^\phi ab \cdot d\theta = \int_\varphi^\phi GZ \cdot d\theta.$

The work done being $W\int_{\varphi}^{\phi} GZ \cdot d\theta$ may therefore be represented by the area of the statical stability diagram between the ordinates corresponding to the angles of inclination φ and ϕ.

If a number of ordinates be taken and the areas of the portions of the diagram from zero up to the successive ordinates be calculated and the values so obtained set up to scale on the corresponding ordinates, a curve of dynamical stability may be drawn through these points such that the *difference* between any two ordinates, say *cd* and *ef*, corresponding to the angles φ and ϕ, would represent the dynamical stability, or work done during the change of inclination, represented by $\phi - \varphi$. Such a curve is represented by *opq* in Fig. 69.

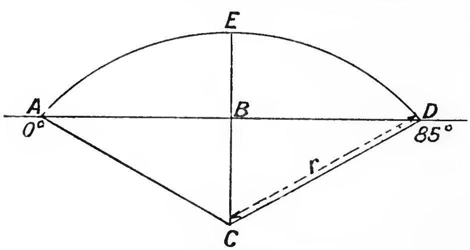


Fig. 69A.

For a complete discussion of dynamical stability of ships the reader is referred to Sir E. J. Reed's work on the "Stability of Ships."

The following is an interesting example :—

The curve of statical stability of a vessel is a segment of a circle of radius twice the ordinate representing the maximum statical stability, which is 2,500 foot-tons. The angle of vanishing stability is 85°. Estimate the total dynamical stability of the vessel.

In Fig. 69A the circular arc AED drawn with centre C represents the curve of statical stability, and is such that CE = 2BE.

It is easily seen that the angle ACD = 120° $\left(=\frac{2\pi}{3}\right)$,

and that $AB = \frac{r\sqrt{3}}{2}$.

$$\therefore \text{ Area of AED} = \frac{2\pi r}{3} \times \frac{r}{2} - \frac{r\sqrt{3}}{2} \times \frac{r}{2} = \cdot 6145 r^2,$$

but $\frac{r}{2} = \frac{2{,}500}{D}$ or $r = \frac{5{,}000}{D}$ where D is the displacement.

$$\therefore \text{ Area AED} = \cdot 6145 \times \sqrt{\frac{5{,}000}{D}}$$

and dynamical stability $= \cdot 6145 \times \sqrt{\frac{5{,}000}{D}}^{2} \times D = \frac{\cdot 6145 \times \sqrt{5{,}000}^{2}}{D}$.

Now the circular measure of $85^\circ = \frac{85\pi}{180} = \frac{187}{126}. = \text{AD}$.

$$\therefore \text{ AB} = \frac{\text{AD}}{2} = \frac{187}{252} = \frac{\sqrt{3}}{2} . r = \frac{\sqrt{3} \times 5{,}000}{2\text{D}}$$

$$\text{or D} = \frac{1 \cdot 732 \times 2{,}500 \times 252}{187}.$$

$$\therefore \text{ Total dynamical stability} = \frac{\cdot 6145 \times \sqrt{5{,}000}^{2} \times 187}{1 \cdot 732 \times 2{,}500 \times 252}$$

$$= 2{,}633 \text{ foot-tons.}$$

The relation between the measurement of dynamical stability and the curve of statical stability provides a means of determining the angle of inclination likely to be reached by a vessel in consequence of the gradual or sudden application of some external force such as wind pressure.

In actual experience it is inevitable that disturbed water would result from or accompany the presence of wind, and the effect of the latter upon the vessel would in consequence be modified. Disturbed water or waves, however, might exist with absence of wind, but in either case a disquieting force would be exerted influencing the inclination of the vessel.

It is not our intention to discuss the question of waves ; to do so would compel an exposition of their formation and other points of external interest, the consideration of which would be outside the purpose of the present work.

It is, however, desirable, and even necessary, to consider concisely the stability—and the manner of determining it—of a vessel under conditions other than that of smooth water ; we shall therefore endeavour to state certain leading features which are capable of facile explanation with reference to the influence of wave water upon the vessel, but before doing so we shall consider the quantitative value of wind pressure in its relation to the stability of a vessel floating in still water.

The effect of wind pressure exerted against the sails or upper works of a vessel will vary according to whether the pressure is a steadily increasing force or is suddenly applied ; in the latter case it may be stated that, roughly, the vessel will heel

over to an angle of magnitude about twice that which would be reached in the former, provided that the ultimate value of the increasing force and that of the force suddenly applied are the same.

Consider a vessel subject to wind pressure which has steadily increased to an intensity p. Let A be the area of the sails or upper works of the vessel against which the pressure is exerted, and h the perpendicular distance between the lines of action of the resultant pressure $p \cdot A$ and the fluid resistance to the motion of the vessel to leeward.

If the vessel were upright the heeling moment due to the wind would be $p \cdot A \cdot h$, but when the inclination θ is reached the moment may be taken as $p \cdot A \cdot h \cos^2\theta$ approximately. Since the wind has been gradually increased, the vessel will have become gradually inclined and will remain steady at the angle θ;

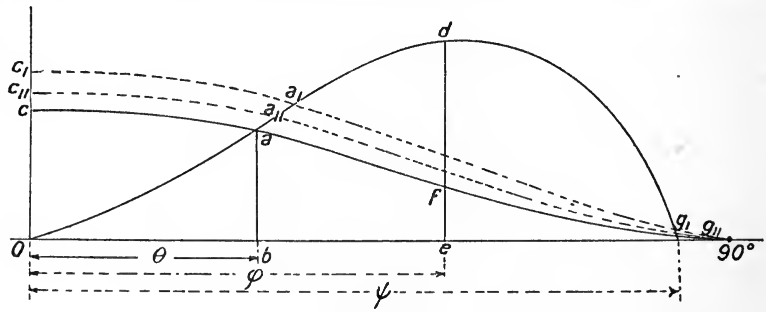


FIG. 69B.

statical equilibrium will result, and the moment due to wind pressure will equal the righting moment of the vessel. This is illustrated in Fig. 69B, where *oad* is the statical stability curve of the vessel, and *ab*—the statical moment at the angle of inclination θ—is also the ordinate ($p \cdot A \cdot h \cos^2\theta$) of the wind curve *caf* for an intensity of pressure p.

Had the wind at this pressure been suddenly applied, an angle of inclination φ greater than θ would have been reached, and the energy expended on the vessel would be equal to $\int_0^\varphi p \cdot A \cdot h \cos^2\theta \cdot d\theta$; the vessel would come to rest momentarily, and the work done by the vessel would be $\int_0^\varphi M \cdot d\theta$, where M denotes the righting moment at any angle of inclination θ.

In consequence of the motion of the vessel she will acquire a certain amount of accumulated or kinetic energy, which will be destroyed by the time she reaches the angle φ, and the condition at any time is such that

$$\int_o^\beta p \,.\, \mathrm{A} \,.\, h \cos {}^2\theta \,.\, d\theta - \int_o^\beta \mathrm{M} \,.\, d\theta = \mathrm{R},$$

where β is any angle the value of which lies between zero and φ, and R is the kinetic energy of the vessel at the angle β.

Analysing the above diagram it will be seen that at the angle θ the area *oca* represents the energy accumulated in the vessel, during which time the righting moment is always less than the heeling moment due to the wind; but beyond θ the righting moment is the greater, and its excess over the moment of wind pressure is used to destroy the energy in the vessel. Obviously the area *adf*—equal to *oca*—represents the energy destroyed, where the energy expended on the vessel up to the momentary position of rest, represented by the area *ocfe*, is equal to the area *oade*, the work done by the vessel during inclination.

From the above consideration it is clear that only the area of that portion of the statical stability curve lying above the wind curve is available for resisting the suddenly applied force, and constitutes the "Reserve Dynamical Stability" of the vessel; such reserve stability is manifestly useful in the event of a vessel being struck by wind gusts or squalls.

If the reserve be considerable it is evident that the angle φ to which the vessel will incline will generally be limited; on the other hand, if the wind pressure or heeling moment due to the same be of so large an amount as to leave a comparatively small margin of dynamical stability, then at no time would the area of the wind curve equal that of the statical stability curve at any specific angle of inclination, and the vessel would capsize; this is illustrated by the wind curve $c_1a_1g_1$ in the above figure, in which the area a_1dg_1 is less than the area oc_1a_1.

The limiting angle ψ to which the vessel may be inclined without capsizing is also shown in the same figure, and is defined by the wind curve $c_{11}a_{11}g_{11}$—drawn for an intensity of pressure p_{11}, say—cutting the statical stability curve at g_{11} such that the area $a_{11}dg_{11}$ is equal to the area $oc_{11}a_{11}$.

Referring again to the case illustrated by the curve *caf*, after the vessel has reached the inclination φ she will tend to return

to the upright against the wind, since the righting moment of the vessel will be in excess of the moment due to the wind until the angle θ is acquired, at which the moments are equal; the vessel now possesses kinetic energy which will carry her to another position of momentary rest at an angle of inclination less than θ, at which this energy will be counteracted by the gradually increasing value of the excess of moment, due to the wind over the righting moment of the vessel, and the resistance of the fluid. The vessel will again move away from the upright, and will continue to oscillate about and ultimately come to rest in the position θ, provided the intensity of the wind pressure has not changed.

Suppose a vessel to be steadily inclined at an angle θ, and that the wind pressure p keeping her at that angle is slowly

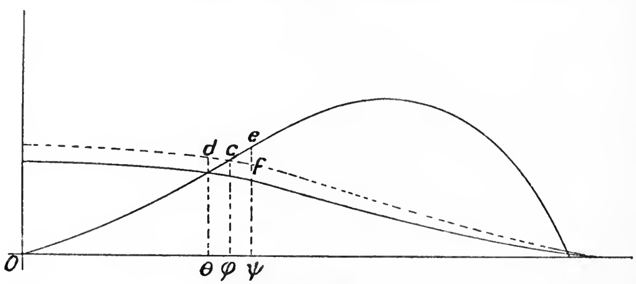


FIG. 69C.

increased to p_1, so that the angle now acquired is φ, then the whole amount of increased work done by the wind is equal to the area *abc* (Fig. 69C), since $ab\varphi\theta$ represents the work done by the initial wind force. But instead of the wind increasing slowly, suppose that it rises suddenly to an intensity p_1 in the form of a squall, the vessel will heel to an angle greater than φ, reaching a momentary position of rest at an inclination ψ_2 say, such that the extra amount of work done by the wind is equal to the additional amount of dynamical stability called into play to resist the inclination; this is illustrated in the figure, where the area *cef* is equal to the area *adc*. Provided the wind remain at this increased pressure p_1, the vessel will return towards the upright, and after oscillating about the position φ come to rest at that angle of inclination.

When we consider the effect of waves on the stability of a

vessel we have to remember the fundamental difference between smooth and wave water, viz., that whereas in the former the weight of the vessel and the resultant fluid pressure act in the vertical direction perpendicular to the surface of the water, in the latter these forces act *generally* in a direction inclined to the vertical and perpendicular to that part of the surface of the wave at which the vessel at the time is floating. We say *generally*, because when the vessel is situated in the hollow or on the crest of the wave the tangent planes to the surface of the wave at these points are horizontal and the lines of action of the above-mentioned forces will be vertical.

Moreover, the buoyancy and weight of the vessel undergo continual change, according to the position occupied relative

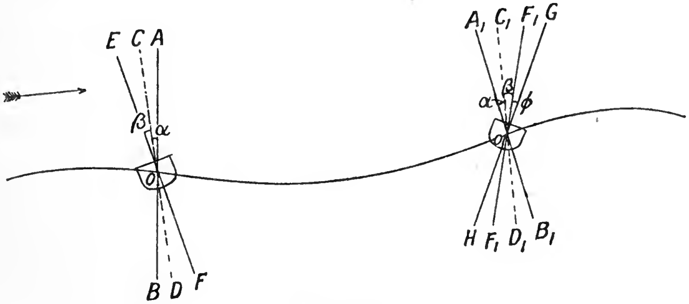


FIG. 69D.

to the wave, their values being least at the crest and greatest in the wave hollow ; the variation, however, is not great and may generally be neglected.

From this it will be seen that if the position of a vessel be such that the line joining the centres of buoyancy and gravity be perpendicular to the wave surface, she will at that instant be subject to no force tending to displace her from that position. For this reason, at any particular position on the wave the perpendicular to the wave surface is considered the virtual upright, and if from any cause the vessel were displaced through an angle θ from the normal to the wave surface, the statical moment of the forces acting upon her due to weight and buoyancy would be the same as if the vessel were inclined through an angle θ in still water ; and therefore the statical stability curve constructed for the vessel under ordinary

conditions of flotation in smooth water may be used when considering the vessel among waves, remembering that in the latter case inclination is reckoned relative to the virtual upright or normal to the wave surface at the point considered, and corresponds with the same angle of inclination to the actual upright in still water.

It is obvious that the inclination of a vessel amongst waves will depend upon the relative natural periods of the vessel and wave, for upon whether the one is greater or less than the other will determine whether the vessel will roll towards or away from the wave slope. If the period of the vessel be small compared with that of the wave the vessel will tend to keep her deck parallel to the surface of the wave, whereas, if the

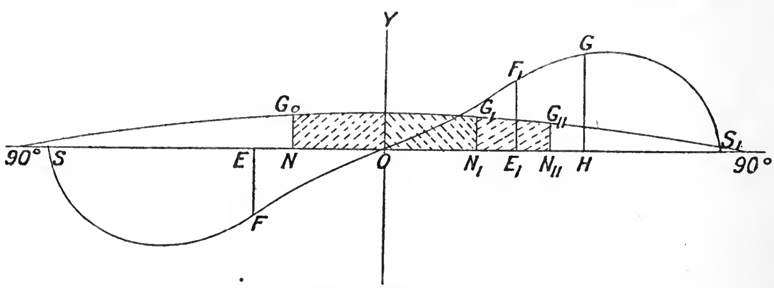


FIG. 69E.

period of the vessel be long compared with the wave period, she will tend to keep upright and her deck horizontal.

Fig. 69D illustrates a vessel rolling into the wave or towards the wave slope through an angle $(\alpha + \beta)$ on each side of the normal AB, A_1B_1 to the wave slope, the latter being inclined at an angle α to the verticals CD,C_1D_1.

In Fig. 69E OF_1S_1 is the curve of statical stability for this vessel, produced for convenience below the line of abscissæ on the negative side of OY, which corresponds with the upright (absolute) condition of the vessel. Set off the ordinates EF, E_1F_1, corresponding to the inclination $(\alpha + \beta)$; they represent the statical moment of the vessel at the time she is inclined at this angle to the normal to the wave slope, and the areas OEF, OE_1F_1 represent the work done by the wave in moving the vessel from the position indicated by EF to the upright and thence to that represented by E_1F_1 respectively.

Suppose wind blowing in the direction of the arrow (Fig. 69D) to strike the vessel suddenly when inclined in the position EF, *i.e.*, at the limit of the inclination to windward.

Draw the wind curve $G_0G_1G_{11}$ in Fig. 69E. The work done by the wind on the vessel during the roll from EF to E_1F_1, *i.e.*, through an angle of 2β, would be practically the same whether the vessel were amongst waves or floating in smooth water. If for the moment we disregard the effect of the angular velocity * of the vessel on the wind pressure, this amount of work would be represented by the area $G_0NG_1N_1$, where $ON = ON_1 = \beta$.

The vessel, however, will move to leeward beyond E_1F_1

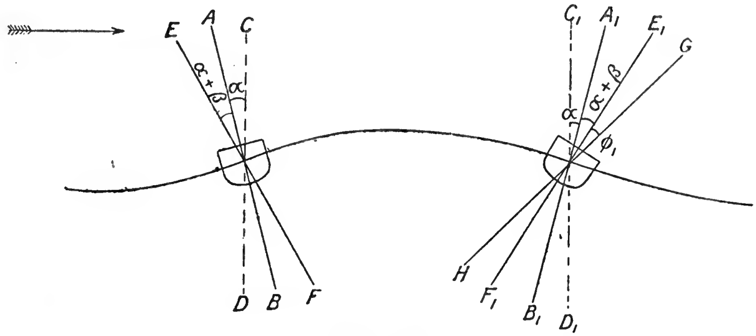


FIG. 69F.

through an angle ϕ to a position GH, determined from the consideration that by the action of the waves alone she would have rolled from EF to E_1F_1; consequently when the position E_1F_1 is reached she has still to expend work equal to the area $G_0NG_1N_1$ due to the wind, and she will therefore continue to roll to GH or through the additional angle ϕ, determined by the condition that the area E_1F_1GH is equal to that of $G_0NG_1N_1$ plus the area $G_1N_1G_{11}N_{11}$, which latter is the work done by

* It is clear that there will be a relative reduction of wind pressure when the vessel is rolling away from the wind and an increase when she is rolling to windward, due to the angular velocity acquired by the vessel, partly by the action of the waves and partly by the wind; as a result the inclination of the vessel to windward will be less than that indicated by EF in the previous figures, while the amount of inclination in the opposite direction will be modified from that indicated by GH, the angle ϕ or ϕ_1 having now a smaller value. This is evident, since now the work done by the wind up to the position indicated by E_1F_1 will be represented by an area less than $G_0NG_1N_1$, and to the limiting position GH by an area less than $G_0NG_{11}N_{11}$.

the wind through a range N_1N_{11} equal to the additional range E_1H, or the angle E_1OG ($= \phi$) (Fig. 69D).

Now consider the vessel to be rolling away from the wave crest through the angle $(\alpha + \beta)$ relative to the normal, as before. She will roll through an angle $(2\,\alpha + \beta)$ relative to the absolute vertical when there is no wind. If she be struck by wind just as she reaches the position EF, she will ultimately roll beyond E_1F_1 to a position GH, and the total angle through which the wind force will be exerted will be $2\,(2\,\alpha + \beta) + \phi_1$, instead of $2\,\beta + \phi$, as in the previous case, where ϕ_1 is different from ϕ.

Appendix to Chapter II.

Amsler's Integrator.

It has been explained (pp. 65 and 81) that the process of determining the statical stability of a vessel for any and various angles of inclination and displacement by applying Attwood's formula is long and tedious, and would engage more time than can be spared in a modern designing office.

The use of Amsler's Integrator is now generally adopted for the purpose of constructing cross-curves of stability from which to determine, under any condition of loading and inclination of the vessel, the statical stability. The method of calculation is described in Chapter II., pp. (82—85), and

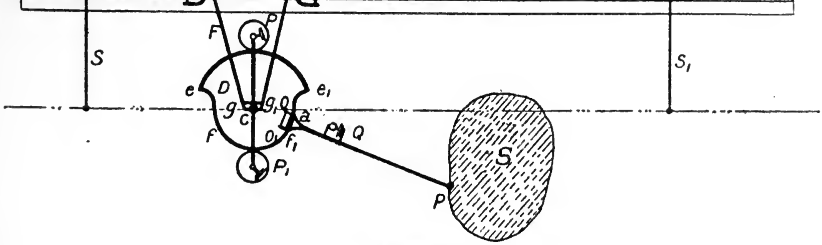


Diagram A.

is such that a smart draughtsman may now complete in a few hours what previously occupied many days.

The instrument, though simple to manipulate, requires careful handling, and is accurate in its readings of areas, moments, and moments of inertia—or more correctly, of functions of these items.

It consists (Diagram A) of a pointer p at the end of an arm ap, which latter works about an axis oo_1 fixed to the disc or frame D. The frame D is finished with two circular arcs ee_1 ff_1, described about a centre c with different radii ; the arcs

have teeth cut upon them, which gear with two pinions P, P_1, and are such that the centres of P and P_1 are always in a straight line with c. To the connecting link or arm joining P and P_1 is attached a light frame F capable of limited rotation about an axis gg_1 at right angles to and passing through the arm PP_1. This frame—and with it the instrument itself—is attached to two wheels w, w_1, for the purpose of giving to the instrument rectilinear motion by causing the wheels to run in a V-shaped groove cut in a long steel batten BB_1, fixed parallel to the straight-line or axis AA_1 passing through C (about which the moments of area and inertia are to be determined) by means of the portable distance pieces s, s_1.

On the radii of the pinions roller wheels are placed, each connected with a counter; the wheels run on the surface upon which the instrument is placed, and are such that the counters record the distances traversed by them.

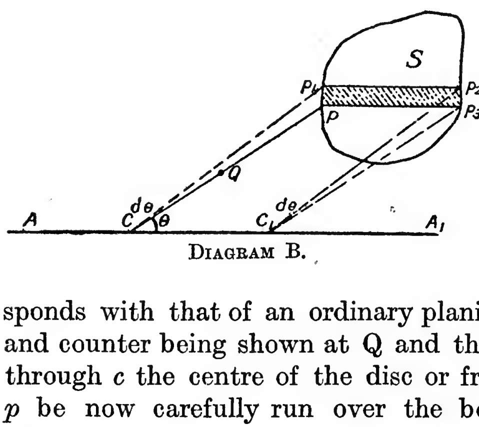


DIAGRAM B.

The arm ap corresponds with that of an ordinary planimeter, the roller wheel and counter being shown at Q and the direction of ap passes through c the centre of the disc or frame D. If the pointer p be now carefully run over the boundary of the area S then :—

The reading of Q will determine the area of S.

" P_1 will determine the moment of the area of S about the axis AA_1.

" P will determine the moment of inertia of S about the axis AA_1.

To prove that the reading of Q gives a function of the area of S,

Let AA_1 represent the axis of the instrument (Diagram B), pQc the arm, of which p is the pointer, c the centre of the disc D, and Q the position of the roller of the planimeter.

Take a strip $pp_1p_2p_3$ of the area S parallel to the axis AA_1 and of indefinitely small width.

The perimeter of the strip may be considered to be traced out by the pointer p by first rotating cp about c through an angle $d\theta$, moving the pointer along p_1p_2 parallel to AA_1 through a distance $p_1p_2 = x$ (say), rotating the arm through an angle $d\theta$ from cp_2 to cp_3, and finally bringing the pointer from p_3 to p (the initial position) along p_3p parallel to AA_1.

Let cQ be denoted by a and cp by b.

The distance moved through by p perpendicular to the arm cp during the motion from

$$\begin{array}{lll} p & \text{to } p_1 & \text{is } -b \cdot d\theta \\ p_1 & ,, \; p_2 & ,, \; +x \sin(\theta + d\theta) \\ p_2 & ,, \; p_3 & ,, \; +b \cdot d\theta \\ p_3 & ,, \; p & ,, \; -x \sin\theta. \end{array}$$

The total distance moved over by the pointer is algebraically $x \left\{\sin(\theta + d\theta) - \sin\theta\right\}$, since $b \,.\, d\theta$ and $-b \,.\, d\theta$ cancel, and this will be the same as that moved over by a point on

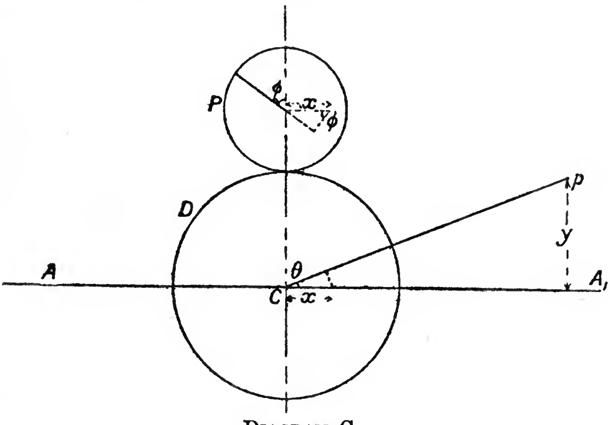


Diagram C.

the circumference of the roller wheel at Q, since the above expression is independent of "b," *i.e.*, the position of the pointer along the arm cp.

Now the area of the strip is given by :—

$$x \left\{b \sin(\theta + d\theta) - b \sin\theta\right\} . = b \,.\, x \left\{\sin(\theta + d\theta) - \sin\theta\right\},$$

i.e., is equal to $b \times$ distance moved through by the pointer.

,, $b \times$ linear rotation of the roller at Q.

The whole area of S may be considered as made up of an infinite number of strips, such as $pp_1\,p_2\,p_3$, and if we take p as

the initial position of the pointer and allow the latter to move over the whole curve continuously in the clockwise direction, ultimately stopping at p, then the "reading of Q," or rather "the difference between the initial and final readings of the counter at Q," multiplied by the length of the arm b will join the area of the curve required.

If the curve has been drawn to any particular scale an allowance must be made in estimating the *actual* area required.

Next consider the pinion P (or P_1) (Diagram C). For a rotation θ of the arm cp (and with it the disc D) the pinion will rotate through an angle $\phi = n\theta + k$ (say), where k is a constant and n is the ratio of the radii of the disc D and the pinion. If c be now moved a distance x along the axis AA_1 we have :—

$$\left.\begin{array}{l}\text{Linear rotation of the roller attached to}\\ \quad\text{the arm } cp\end{array}\right\} = x \sin \theta,$$

$$\left.\begin{array}{l}\text{Linear rotation of the roller attached to}\\ \quad\text{the pinion}\end{array}\right\} = x \cos (n\theta + k),$$

or the relation of the simultaneous readings of the planimeter and pinion is as $\sin \theta$ to $\cos (n\theta + k)$.

If $n = 2$ and $k = 0$ the relation becomes

$$\sin \theta \text{ to } \cos 2\,\theta = 1 - 2 \sin^2\theta,$$

$$i.e., \frac{y}{b} \text{ to } 1 - \frac{2y^2}{b^2}.$$

If, as before, we consider a strip $pp_1p_2p_3$, and let $\frac{y_1}{b} = \sin (\theta + d\theta)$, then the ratio of :—

Difference between the initial and final readings of Pinion
to
Difference between the initial and final readings of Q

$$= \frac{2}{b}(y_1 + y),$$

$$\begin{aligned}\text{or index reading at pinion} &= \bar{y} \times \text{function of area of strip},\\ &= \text{function of moment of strip},\end{aligned}$$

$$\text{where } \bar{y} = \frac{y_1 + y}{2}$$

and therefore for the complete area S we have :—

$$\begin{aligned}\text{Index reading at pinion} &= \int \text{function of moment of strip}\\ &= \text{function of moment of S}.\end{aligned}$$

Again, if $n = 3$ and $k = -\frac{\pi}{2}$ the above relation would become :—

$$\sin \theta \text{ to } \cos\left(3\theta - \frac{\pi}{2}\right) = \sin 3\theta = 3 \sin \theta - 4 \sin^3\theta$$

$$i.e., \frac{y}{b} \text{ to } \frac{3y}{b} - \frac{4y^3}{b^3}.$$

And, using a similar argument, it will be seen that the index of the pinion would represent the difference between a function of the moment of inertia of the area S and a function of that area. In practice the necessary subtraction has to be performed in order to reduce the index reading of the pinion to a function of the moment of inertia of the area ; while the moment of area and the area itself are obtained direct from the instrument.

In Amsler's Integrator the radii of the arcs of the disc D are as 2 : 3, and the radius of either pinion to that of the smaller arc as 1 : 2, from which it will be clear that the moment readings are obtained from P_1 and the moment of inertia readings from P.

In determining cross-curves of stability by the aid of Amsler's Integrator it is only necessary to use the *area* and *moment of area* readings of the instrument ; the readings for the moments of inertia not being required.

The method of application of the integrator to the lines of a vessel in stability calculations was described on p. 82, and it was there assumed that the transverse or vertical sections of the vessel were spaced according to one of Simpson's Rules.

Tchebycheff's Rules.

A method known as Tchebycheff's Rule is now often employed in ship calculations, and while it is considered to be sufficiently accurate, the amount of labour involved is considerably less than when the computation by Simpson's Rules is made.

By this method the ordinates or sections chosen are less in number than when Simpson's Rules are used, and they are so spaced that by simply adding them together and dividing by the number of ordinates we get the mean or average ordinate or section, the value of which when multiplied by

the length of the base gives the required area or volume, as the case may be.

In making a stability calculation it is clear that if the vertical sections of the vessel be taken at intervals, spaced according to Tchebycheff's Rules, we may run the pointer of the integrator over the whole of the sections in succession instead of over the odd, even, and half-sections separately, as is necessary with a spacing to Simpson's Rules. The difference between the initial and final readings, divided by the number of sections and multiplied by the length of base (*i.e.*, the length of water line) and the constant for the instrument, gives the volume of displacement. The lever or arm for stability may be obtained by dividing the difference of the *moment* readings by the difference of the *area* readings and multiplying the quotient by the constant of the instrument.

The rule for the spacing of the ordinates or sections is indicated in the following table :—

Tchebycheff's Spacing of Ordinates.

POSITION OF ORDINATES RELATIVE TO MIDDLE OF BASE GIVEN IN TERMS OF HALF-LENGTH OF BASE.

No. of Ordinates.					
2.	·5773				
3.	0	·7071			
4.	·1876	·7947			
5.	0	·3745	·8325		
6.	·2666	·4225	·8662		
7.	0	·3239	·5297	·8839	
8.	·1026	·4002	·5938	·8974	
9.	0	·1679	·5288	·6010	·9116
10.	·0838	·3127	·5000	·6783	·9162.

The proof of the rule is not so rigorous as in the case of Simpson's Rules. It is founded on the assumption that the curve, the area of which is required, is of parabolic form and represented by the general equation

$$y = a_0 + a_1 x + a_2 x^2 + a_3 x^3 +, \text{etc.},$$

and that the degree of the equation will depend upon the number of ordinates to be chosen.

For example, suppose two ordinates (Diagram D) are to be taken, the equation of the curve would be :—

$$y = a_0 + a_1 x + a_2 x^2.$$

The area of the curve is

$$\int_{-l}^{l} y \,.\, dx = 2l\left(a_0 + \frac{a_1}{2} \,.\, l + \frac{a_2}{3} \,.\, l^2\right) \quad . \quad . \quad . \quad \text{(a)}$$

But according to the above hypothesis—

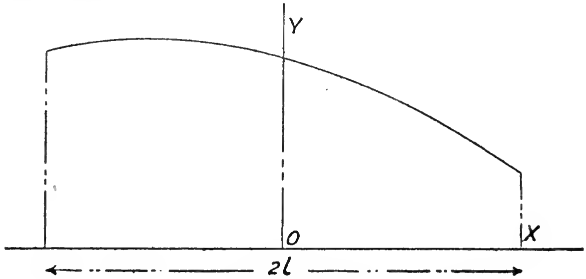


DIAGRAM D.

Area $= k \times$ sum of the two ordinates (y_1 and y_2, say), where k is a constant to be determined;

now when $x = x_1$, $y_1 = a_0 + a_1x_1 + a_2x_1{}^2$

" " $x = -x_1\ (= x_2)$, $y_2 = a_0 - a_1x_1 + a_2x_1{}^2$

$\therefore\ y_1 + y_2 = 2a_0 + 2a_2x_1{}^2 = 2\,(a_0 + a_2x_1{}^2)$

$\therefore$ Area $= 2k\,(a_0 + a_2x_1{}^2)$ . . . . . (b)

Equating the coefficients of a_0, a_1, etc., in (a) and (b) we have :—

$$2k = 2l, \text{ or } k = l.$$

$$2kx_1{}^2 = \frac{2l^3}{3}, \text{ or } x_1{}^2 = \frac{l^2}{3}, \text{ or } x_1 = \pm \cdot 5773l:$$

i.e., the ordinates are situated at a distance of ·5773 × half-length of base, on either side of the centre of base.

A similar solution would be obtained for a number of ordinates greater than two. For eight and ten ordinates, however, imaginary roots are obtained and the solution fails, but these ordinates may be determined by considering the curve on either side of the middle ordinate and treating each of these portions for four or five ordinates as the case may be; in this way we are able to determine the position of the ordinates as indicated in the above table.

CHAPTER III

SUBMARINES

Unlike the ordinary vessel with which we are so familiar, the stability of which formed the subject of the preceding chapter, the submarine, as the term implies, finds her sphere of activity essentially below the surface of the water. At times it is necessary for her to appear at the surface—at least for a short period—in order to take bearings or otherwise to execute some necessary evolution. When not actively engaged she will of course remain moored at the surface in readiness to embark her crew, stores, etc., preparatory to further active service.

In considering the subject of the stability of submarines, therefore, it is clear that both the surface floating and the submerged conditions of the vessel will have to be noted.

The statical stability of a vessel floating under ordinary conditions at the surface will generally suffer modification as the draught increases, *i.e.*, as the draught approaches the depth of the vessel, until, when the deck edge becomes awash or the reserve buoyancy is *nil*, the metacentre coincides with the centre of buoyancy; the distance between the centre of gravity and centre of buoyancy is then the metacentric height for this—upright—condition. If the vessel be now entirely submerged the value of the initial stability remains unchanged and represents the maximum righting arm for the submerged condition.

For any angle of inclination θ the righting arm becomes HG sin θ, reaching the maximum value at an angle of inclination of 90° were it possible for the vessel to heel over to this extent without internal derangement.

With reference to angular movement in the longitudinal direction, a vessel possessing satisfactory trim and the ability to steer correctly when floating at the surface might be found not to behave so well when submerged; the change in condition

results in a modification of the factors governing her angular movements in this direction; the vessel is found to be more sensitive to an alteration in the distribution of the weights in the fore-and-aft direction, such, for example, as the movement of a man on board, the discharge of a torpedo, or the addition or ejection of water ballast; and the longitudinal stability might be so affected thereby as to cause her to dip by the head or the stern, as the case may be, and to steer erratically in the longitudinal vertical plane. These were real difficulties which had to be encountered when the submarine was more

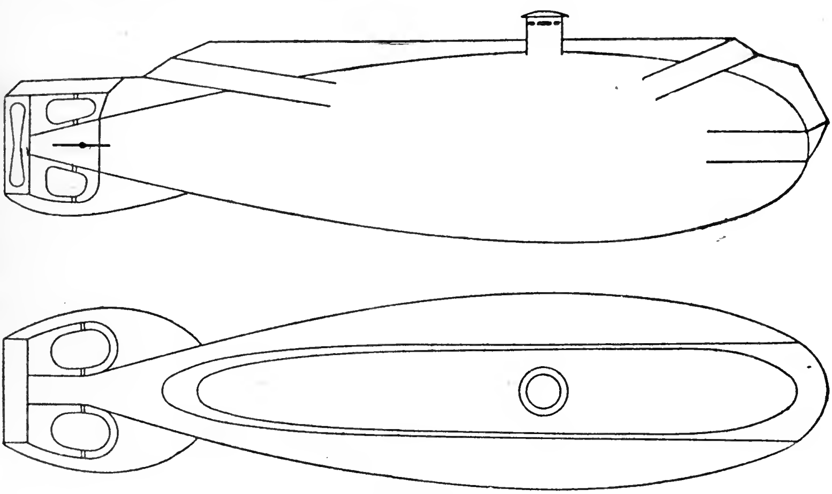


Fig. 70.

or less in the early stages of development, and in consequence the stability of the submarine presented a very serious problem.

While both the transverse and longitudinal stability are of the utmost importance, it is evident that the latter presents the greater difficulty, but in every case it is essential that the safety of the vessel and its efficiency as a reliable fighting machine be assured in respect of angular displacements in both the transverse and longitudinal direction.

The early difficulties have now practically been overcome, and we find that under the care of a skilful commander the functions for which the vessel is designed may now be performed with comparative ease and precision.

It is not intended here to enter into an account of the development of the design of the submarine vessel, nor of its construction; this information may be gleaned with a great amount of profit from those published works * dealing with the history and development of submarine navigation.

A few words on "form," however, are necessary, as with this the question of stability is naturally closely allied.

In its inception various fantastic forms of the submarine were introduced, culminating, it may be said, in the cigar shape, which latter eventually gave place to the fish form with bluff nose and a transverse section practically circular, as exemplified in the Holland boat as we know it to-day (Fig. 70).

There has been no sudden transition of form, but, like most of the changes which have been wrought in marine design, the development has been of a tentative nature, and as the result of much labour, experiment, and monetary outlay the present form has been evolved.

The type of vessel is unique; functions have to be fulfilled which clearly are outside the requirements of an ordinary vessel, and which, moreover, were previously unknown to navigation. Not only has the vessel to perform work at the surface but beneath it; she has to make her course with the compass for guide, preserving a definite angle of inclination to the horizontal free from oscillation and retaining her stability.

It is easy to submerge a vessel, but to do so and preserve a specific depth below the surface is a problem of some difficulty, and one which even now may be said not to have been solved altogether satisfactorily.

The question of submersion naturally hinges on the relative values of the weight and buoyancy of the vessel, whether she be floating at or suspended below the surface. Some of the methods by which submersion or flotation, as the case may be, has been obtained may be noted.

In the case of the *Nautilus* the vessel was made to rise or sink by pushing outward or withdrawing respectively pistons working in cylinders in the sides of the vessel.

In other cases the vessels were fitted with portable weights or drop-keels, which enabled them to sink and which when

* *E.g.*, Burgoyne's "Submarine Navigation."

released allowed them to rise quickly to the surface, often, as would be supposed, with not inconsiderable speed, and with some degree of inconvenience to the human element on board and the probable loss of the released weights.

Some vessels have been designed with buoyancy *nil*, but no great amount of success has been realised with them. They are naturally very tender and sensitive to very slight disturbances, whether those disturbances be of the nature of surface-water disturbance or the movement of men or weights on board. Even small quantities of water finding ingress into ballast tanks, perhaps through the agency of leaky valves or otherwise, would at once create a negative buoyancy and cause the vessel to sink lower and lower—possibly unrealised by the men on board—thereby jeopardising her safety.

Conversely, by the ejection of a small quantity of water, a positive buoyancy would be established, causing, or tending to cause, the vessel to rise, and so such a vessel would be more or less continually in a state of vertical oscillation detrimental to its usefulness.

Submarines now generally have a reserve buoyancy of a few hundred pounds, and the approved methods of obtaining trim or longitudinal inclination, causing the vessel to sink or dive and preserving a definite required depth below the surface, consist in the use of water ballast on the one hand and the horizontal rudder on the other, while the vertical rudder is used for steering in the horizontal plane in a manner similar to that known in the case of ordinary vessels.

One other case of positive buoyancy may be mentioned in which vertical propellers, *i.e.*, propellers working on vertical shafting, were used to bring the vessel below the surface, the action of the propellers neutralising the excess of buoyancy over weight. There was the difficulty, however, of preserving the exact desired depth, similar to that experienced with the vessel of buoyancy *nil* already described ; the idea was either to reduce or increase the revolutions of the propellers according as the vessel was required to rise or sink, but the vertical oscillatory trouble was always apparent.

The horizontal rudder as the one means of obtaining submergence and of preserving the vessel in that condition on a steady course has become established ; the boat is trimmed by

means of water ballast tanks located at the extremities, and the change of draught from the light to that approaching the awash or diving condition is assisted by water ballast situated amidships.

It is clear that the position of the centre of gravity of the vessel complete and in diving condition must be preserved unchanged except in so far as it may be necessary to bring about a change at the will of the commander, as, for example, by firing a torpedo, the effect of which would be estimated beforehand and allowed for.

In all cases, however, the ballast tanks and all compartments used ordinarily for containing liquids must be either *completely filled* or *completely empty*, in order that one possible source of danger, viz., the presence of liquid free to move, may be eliminated ; by these precautions the precise position of the centre of gravity is calculable, and for good stability is kept comparatively low.

In order to dive the vessel is propelled forward, and at the same time the horizontal rudder is brought into action to depress the nose or bow. Generally under surface conditions the bow may be assumed to be initially submerged, and if the vessel be started ahead there is an inherent tendency to dip by the head without the aid of the horizontal rudder, and with the broad-nosed modern boats this tendency is enhanced.

It is therefore necessary to exercise care in giving speed to a submarine when in the surface condition, because if a certain speed—depending on the form of the boat—be exceeded she may be forced under although the horizontal rudder has not been used, with possible dire results should she happen not to be sealed.

The boat will dip only on a certain speed being attained. For a less speed the change of distribution of fluid pressure consequent on that speed as the boat is forced ahead will not suffice to cause the bow to take the necessary depression. In the case of the Holland boats the minimum speed necessary to keep the vessel submerged is about five knots.

Hydroplanes or fins acting as adjuncts or auxiliaries to the horizontal rudder have been introduced with good results. They assist the rudder in its action, and possess good steadying qualities when placed symmetrically on the vessel. One of their functions is to prevent the tendency of the vessel to *dive*

when the horizontal rudder, which is situated right aft, is put down. This tendency, as has been explained above, is always present in the full-nosed vessel as soon as she is given speed, and the forward hydroplanes may be so placed as to experience an upward fluid pressure in order to counteract this downward tendency at the bows. The objection to hydroplanes is resistance to speed.

It is evident that the efficiency of the Holland and up-to-date types of submarine is obtained in a way quite different to that in the case of the earlier types, inasmuch as the modern boat depends entirely upon her rudders, the power of which in turn

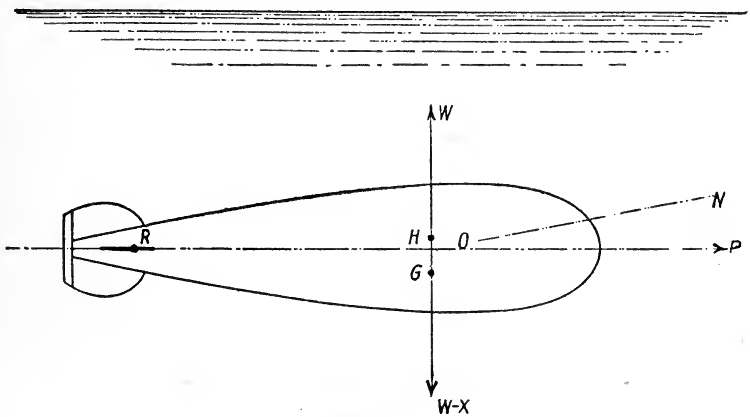


FIG. 71.

depends upon the speed of the vessel ; in consequence of this these boats have been well designated the *dynamic type.*

It may not immediately be clear that for a submerged submarine to proceed in a horizontal line or plane it is necessary that she be propelled downwards, the amount of depression or angle of inclination to the horizontal being dependent upon the speed at which she is driven. This will be seen by considering Fig. 71.

Imagine the vessel while proceeding below the surface to be placed on even keel and this condition maintained with the horizontal rudder R placed horizontally.

Let P denote the propelling force in the horizontal direction, W — X the weight of the vessel, and W the buoyancy acting vertically. The resultant vertical force X acting upwards

combined with P gives a resultant in a direction inclined upwards, as indicated by ON in the figure, along which the vessel will proceed towards the surface. The opposing force is the fluid resistance, and upon the relative value of this latter and the resultant of X and P will depend the time taken by the vessel to reach the surface.

A similar case is that of a projectile travelling through the atmosphere ; so long as there is a resultant force in the vertical

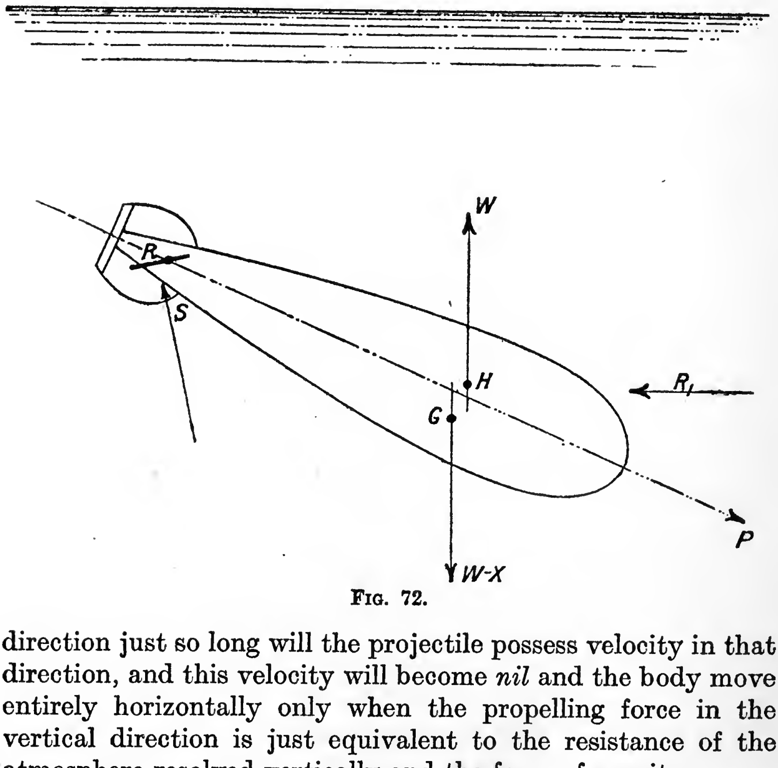


FIG. 72.

direction just so long will the projectile possess velocity in that direction, and this velocity will become *nil* and the body move entirely horizontally only when the propelling force in the vertical direction is just equivalent to the resistance of the atmosphere resolved vertically and the force of gravity.

For the vessel, then, to move horizontally the horizontal rudder must be put down at least through a small angle, in which case the disposition of the forces acting on the vessel will be as indicated in Fig. 72, where

P denotes the propelling force.
R_1 the resistance to motion.
S the fluid pressure on the rudder.

If we assume the vessel to be travelling in the horizontal direction with uniform speed, we have :—

The resolved part of P horizontally is equal to the sum of the resolved parts of R_1 and S in the same direction.

Also the sum of the resolved parts of P, R_1 and S in the vertical direction together with W and W — X is zero.

Further, the sum of the moments of all the forces about some horizontal transverse axis is zero.

These are the conditions for uniform speed horizontally,

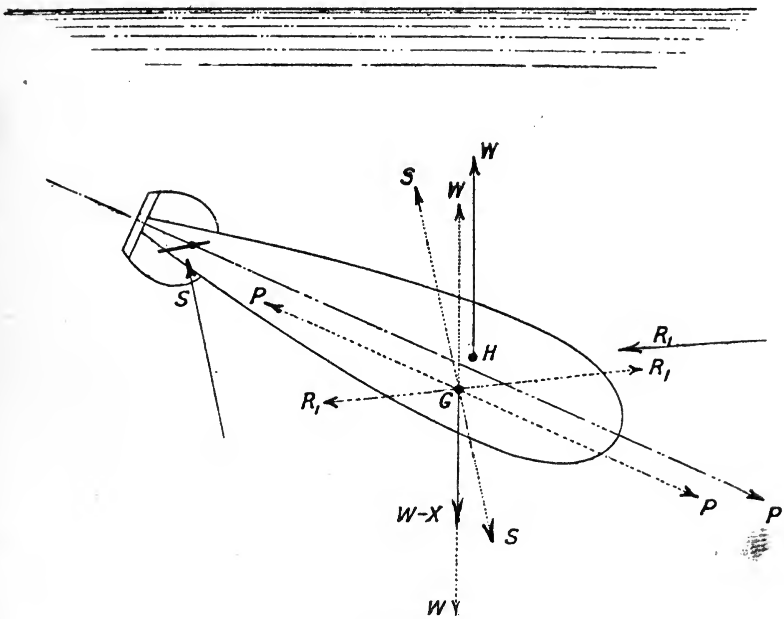


FIG. 73.

absence of motion or oscillation vertically, and no rotation in the vertical plane.

The position of the axis about which the vessel will momentarily turn if the balance of the forces be disturbed is generally assumed to be situated at or near the centre of gravity G of the vessel. Its exact position, however, is difficult to estimate, due to the uncertainty of calculating the value and direction of all the acting forces. For this reason the adjustment of the rudder to suit any particular condition of running must necessarily be obtained by experiment, or tentatively with the vessel herself,

the helmsman being guided by the indications of a manometer in direct communication with the sea in determining whether the vessel is running at a certain required depth below the surface or not, in which latter case he must modify the angle of inclination of the horizontal rudder.

Suppose the system of forces acting on the vessel at any moment to be as represented in Fig. 72. The condition will not be affected if we apply at G forces equal to, and their

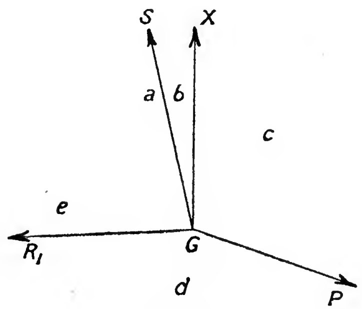


Fig. 74.

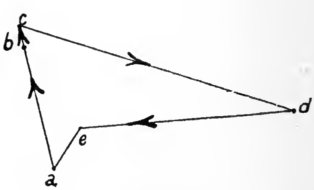


Fig. 75.

directions the same and opposite to, those of the acting forces. The added forces are illustrated in Fig. 73 by the broken lines.

Let d_s, d_p, d_r, and d_w be the perpendicular distances of G from the directions of the forces S,P,R_1 and W respectively.

It will be seen that the system of forces may now be resolved into—

(*a*) A couple, the value of which is given by :—

$$S \times d_s + P \times d_p - R_1 \times d_r - W \times d_w = Q \text{ (say)}$$

and (*b*) The forces represented in Fig. 74 acting at G. Constructing a diagram of forces Fig. 75 we obtain "*ae*," representing the resultant force acting through G.

The system of acting forces has therefore been resolved into a couple Q and a force represented in magnitude and direction by "*ae*" acting through G.

For a positive value of Q the vessel will tend to dip by the head, and conversely a negative value of Q will tend to raise the nose of the vessel, while the action of the force "*ae*" is to cause the vessel to rise or fall according as its direction is as indicated in Fig. 75 or the opposite respectively.

If Q be zero and "*ae*" be also zero, the system of forces balance and the vessel will proceed in a straight line. If under such a condition the balance were disturbed there would result a tendency to rotation, which would cause the vessel to rise or dive, as the case may be, and the particular value of Q would represent this tendency. That part of Q represented by the couple $W \times d_w$ is the inherent longitudinal stability of the vessel.

It is quite evident that there is a very intimate association between the form of the vessel and the value assumed by Q at any moment, but the final factor governing the value of Q is contained in the ability of the helmsman to so manipulate the steering gear as to keep the vessel on the desired course free from oscillation.

The blunt nose and fine after end is now practically accepted as giving the vessel the necessary good steering qualities, favouring speed and minimising oscillation, points all of which

FIG. 76.

are most essential in the case of a vessel which has such unique functions to perform, and the absence of which might easily spell disaster, not only to the vessel herself, but to those other units of the fleet with which she might be co-operating. This is at once apparent when we consider that small variations of steering in the vertical plane are likely to bring the vessel nearer to, or take her farther from, the surface than is necessary for the successful carrying out of the project in hand ; whereas even a small amount of oscillation would bring about inaccuracy of aim in firing a projectile.

By careful manipulation of the steering mechanism the oscillation may be reduced to a minimum and the vessel kept on her course, not perhaps in a straight line, but in a path partaking more or less of a wave form, such as is indicated in Fig. 76, in which the amplitude is small compared with the length of the wave profile, the mean straight path being preserved.

From the consideration of longitudinal stability and angular displacement in the fore-and-aft direction we will now turn to the question of transverse stability.

Being essentially a diving boat, the vessel must possess a certain amount of surplus buoyancy, so that although the principal function is to act below, yet when occasion requires it and under ordinary conditions she must be able to float at the surface. She must therefore be such that the total weight of hull, machinery, and equipment—termed the light condition—is less than the weight of water of volume equal to that of the vessel; and not only so, but when fully manned, with all stores, etc., on board, and in fighting condition, she must still possess an amount of reserve buoyancy sufficient to allow her to be sealed up before diving. This is the *diving* condition, in which there is generally a margin of a few hundred pounds reserve buoyancy.

There are then three principal conditions requiring special attention, two of which refer to flotation, viz., the *light* and *diving* conditions, and the third to *submergence*, this latter only differing from the diving condition in so far as it is brought into being by the action of the propeller and the horizontal rudder.

When floating with any portion of the upper works exposed, as in the first two conditions cited above, the submarine is subject to the same laws as regards stability as the ordinary vessel, and even when submerged there is a similarity of treatment with reference to transverse stability.

The illustrations shown in Fig. 77 represent the transverse section of a submarine—not necessarily circular—in the upright and inclined positions, floating at the water line LL_1. G and H indicate the centres of gravity and buoyancy respectively, situated in the same vertical line when the body is at rest in the upright position. When the vessel is inclined the position of H changes to H_1, and the vertical line through H_1 intersects GH in the point M, which for small inclinations is the metacentre. The righting couple tending to restore equilibrium is $(W - X_1)GZ$, where GZ is the perpendicular drawn from G on to H_1M.

The principle upon which the stability is based is therefore identical with that relating to the ordinary vessel, but we have to bear in mind that whereas in the latter the water plane is invariably of appreciable area, in one (the *diving*) condition of the submarine the water plane area is reduced to a minimum and is very little more than the sectional area of the conning-tower.

From this consideration it will be seen that the value of HM, which is equal to

$$\frac{\text{Transverse moment of inertia of water plane area}}{\text{Volume of displacement}}$$

is under ordinary conditions not likely to be large and approaches zero as the diving condition is reached, which value it assumes when the vessel becomes submerged and the metacentre and centre of buoyancy coincident.

Imagine the vessel submerged and inclined at an angle θ to the upright; then, considering the action of W and W $-$ X only, these forces may be resolved into the couple $W \times HG \sin \theta$,

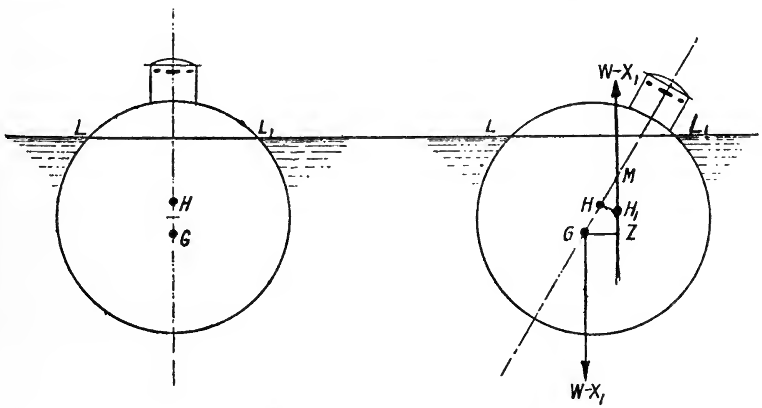


Fig. 77.

tending to restore the vessel to the upright position, and a vertical force acting upwards at G equal to X.

Consider now the remaining forces acting on the vessel, viz., S,R_1 and P (see Fig. 73), all of which may be considered to act in the longitudinal plane of symmetry containing HG, and therefore the axis—assumed to be situated at (or near) G—about which the vessel is rotating.

Combined they constitute a resultant oo_1 (Fig. 78), tending to cause angular movement in the longitudinal plane HG and motion of translation in that plane in a manner similar to that explained on p. 176, but no angular movement transversely. For no motion of the vessel in the vertical direction the resolved part of oo_1 vertically must counteract X, and so far as transverse angular movement is concerned, therefore,

we are interested only with the couple $W \times HG \sin\theta$, which constitutes the statical stability of the vessel transversely. Should the axis of rotation not be situated at G, then the restoring couple would be given by

$$W \times HG \sin\theta \mp X \times d,$$

according as the axis is situated between or outside the verticals through H and G, where "d" is the perpendicular distance of the axis from the vertical through G.

It may generally be assumed that "d" is small and that the second quantity, viz., $X \times d$, in the above expression may be neglected.

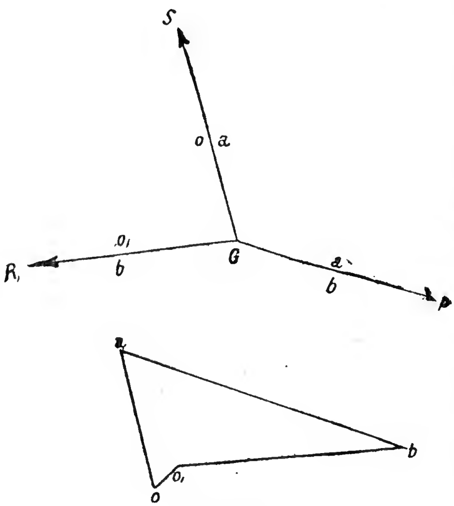


Fig. 78.

Brief reference will now be made to the effect of the vertical rudder upon the transverse stability of the submarine.

In the case of the ordinary vessel it is not usual except in certain instances—as, for example, with torpedo-boat destroyers and other small vessels of high speed—to consider the heeling effect of the vertical rudder, but in the case of the submarine it forms a subject of some importance.

When the rudder is put over, the fluid pressure upon it at once causes or tends to cause the vessel to pursue a curvilinear path ; this pressure may be resolved into :—

(*a*) A component in the fore-and-aft direction, tending to retard the progress of the vessel ;

(*b*) A component, Q, acting laterally or perpendicular to the longitudinal plane through the centre of pressure of the rudder.

In addition to Q we have the fluid resistance, T, to centrifugal action, and the centrifugal force

$$\frac{(W - X) \times V^2}{g \times R_2},$$

where V is the speed with which the vessel is moving and R_2 the radius of curvature of the path at the particular instant considered, this force acting outwards through the centre of gravity of the vessel.

These forces, viz., Q, T, and $\frac{(W-X)V^2}{g \times R_2}$, will act together to influence the heeling of the vessel as she leaves the straight line for the curvilinear path.

Fig. 79 shows diagrammatically the system of transverse forces acting on the vessel in turning when the helm is put over. Upon the disposition of these forces will depend the angle to

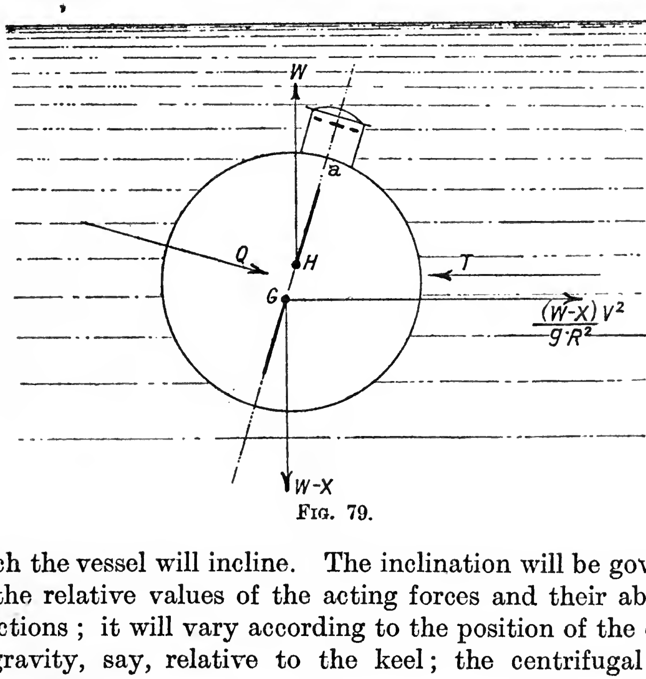


FIG. 79.

which the vessel will incline. The inclination will be governed by the relative values of the acting forces and their absolute directions; it will vary according to the position of the centre of gravity, say, relative to the keel; the centrifugal force $\frac{(W-X)V^2}{g \cdot R^2}$ acting through this point.

Generally the vessel will incline inwards or outwards, according to whether the centre of gravity occupies a low or high position in the vessel, while for some intermediate position of the centre of gravity the vessel will remain upright—dependent upon the value of V.

CHAPTER IV

FLOATING DOCKS

When dealing with the stability of bodies of ship-shape form (Chapter II.) we had occasion, p. 54, by way of illustrating the principles of stability, to take the case of the simple body of rectangular shape.

The present chapter treats of floating docks—bodies for the most part of rectangular shape—the stability of which will require to be considered under new conditions. Not only will it be necessary to consider the dock floating alone under its own weight, but also with the vessel for the docking of which it is designed resting on the blocks.

Three types of dock will be considered, viz. :—

The double-wall or U dock.

The offshore—usually designated L—dock.

The outrigger or depositing dock (L dock combined with an outrigger pontoon).

The points of difference in the calculation of the stability of floating docks as compared with that of an ordinary ship-shape vessel may be summarised as follows :—

(1) The water plane area and the moment of inertia of the same necessarily vary considerably between the deep draught at which the vessel takes the blocks and the least draught at which the dock floats with the pontoon deck entirely above water. Between these limits we must consider the effect of the variation of the water plane area and the volume of displacement of the vessel.

(2) The compartments of the dock will contain water which, should the dock become inclined from the upright, will be free to move. With a ship such free bodies are usually assumed to be non-existent when stability calculations are made, *i.e.*, water ballast or other compartments constructed for carrying fluids are assumed to be either full or empty.

(3) The effect of wind pressure may be such that in the case of a ship free to move considerable heeling will result. With the dock, however, *practically* no heeling is permissible during the process of docking, and thereafter, as any appreciable tendency to incline after the vessel had taken the blocks would bring undesirable straining action upon the shores and supports with possible disastrous consequences.

Therefore in the ordinary case it is necessary only to consider *small* angles of heel up to one or one and a half degrees, and the

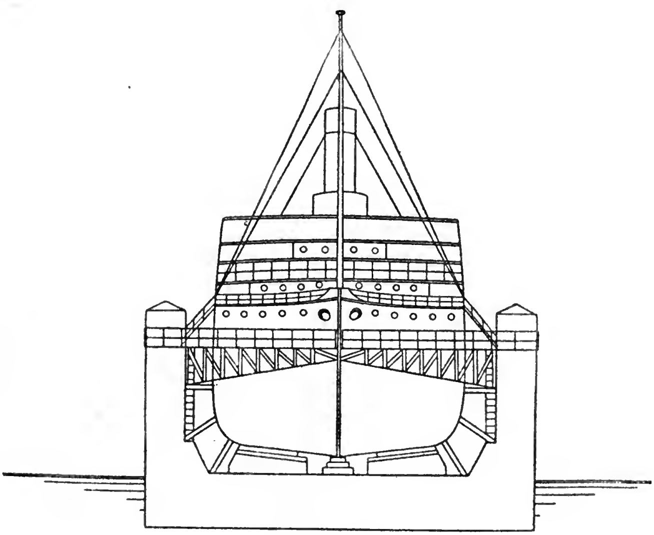


Fig. 80.

calculations must be made at intervals from the minimum to the deep draught.

The case of the double-wall dock—a general view of which is shown in Fig. 80—presents the simplest problem, for the obvious reason that the structure is symmetrical about the centre longitudinal plane and floats practically free from constraint except for its moorings. With the vessel resting on the blocks the system, *i.e.*, the dock with the vessel, is still one of symmetry with reference to the middle line plane of the dock, and the water plane area, though broken, has its centre of area in this, the middle line, plane ; and the moment of inertia of

the water plane area will be considered about the longitudinal axis passing through this point.

As the admissible angles of heel are very small the metacentric method of stability is generally sufficient, *i.e.*, it is only necessary to consider the relative position of the " centre of gravity " and the " metacentre " of the dock or system for the upright position, the calculation being made for a series of draught lines between the minimum and maximum draughts to which the dock is to be designed.

Where the dock when built is required to be towed across the sea to its ultimate destination, or where careening for repairs or painting is a condition of the design, a further calculation for *statical* stability would be made. Where there is a break in the continuity of the structure, or an abrupt change in the design, it is necessary to specially determine the stability at the draughts corresponding to these points of discontinuity or change. Such points exist at openings in the dock sides or walls, the deck of the pontoon, and at the top of the keel blocks.

In submerging, or raising the dock, the interval occupied from the time at which the pontoon deck is awash until the water reaches the upper surface of the keel blocks, and *vice versâ*, has been designated the *critical* period.*

During this period—whether a vessel is resting on the blocks or not—the stability is determined entirely by the dock itself so far as the area and moment of inertia of water plane and the displacement are concerned.

If we consider the dock and vessel together it is evident that whereas the centre of buoyancy is situated low down, the centre of gravity of the system is comparatively high, and therefore in order to ensure stability it is necessary to widen the walls at the bottom in order to bring the metacentre sufficiently high above the centre of gravity ; as regards the dock itself—*i.e.*, without the vessel resting on the blocks—the centre of gravity is situated relatively much lower and the argument for raising the position of the metacentre is not so potent.

It is for the reasons stated above that in the case of the U dock the walls are made fairly broad for a height at least

* Mr. Lyonel Clark on " Floating Docks," Proceedings I. C. E., Vol. 161.

represented by that of the keel blocks, and in the case of the L dock an air box is built on the outer edge of the pontoon to about the same height or a little higher, in order to augment the stability during the critical period.

Contrary to the case of the U dock, the centre of area, and therefore the axis about which the moment of inertia of water plane is taken, will, in the case of the L dock, not lie in the longitudinal plane passing through the centre of the blocks; its position will be governed by the horizontal sectional area of the dock wall, and of this latter in conjunction with the water plane area of the ship docked; as this area will vary considerably as the draught is altered so will the *common* centre of area of water plane take up a lateral position variable with the draught.

Assuming the volume of fluid displaced to remain constant as the dock or system is inclined, and that the new centre of buoyancy is situated in the transverse plane containing the centre of buoyancy and centre of gravity for the upright position, then the intersection of the new and the initial water plane will pass through the common centre of area of water plane as defined above, and the moment of inertia of water plane area will be taken about this line of intersection.

The positions of the "centre of buoyancy" and "centre of gravity" require to be carefully calculated.

With reference to the centre of buoyancy, it is evident that not only will it take up a position in the vertical direction variable with the draught, but in the case of the L dock the lateral position will vary also, due not only to the unsymmetrical character of the structure of the dock itself, but also to that of the system when the dock supports a vessel on the blocks.

Unlike the ship-shape body and the submarine already considered, the centre of gravity both in the case of the dock only and the system (dock and ship combined) will vary in position as the draught changes in consequence of the water which enters or is pumped out of the dock.

For these reasons it is better to draw the actual loci of the "centre of buoyancy" and "centre of gravity" * relative

* The locus of centre of gravity could be only approximately drawn, due to the uncertainty of the distribution of water in the dock at any time during the process of docking or undocking.

to the dock itself, bearing in mind that for equilibrium the line joining these two points corresponding to any particular draught must be vertical.

The free water contained within the compartment will generally have a negative influence upon the metacentric height, as explained in Chapter I., p. 29, the effect being measured, for any one compartment, by the moment of inertia of the area of free water surface about a longitudinal axis through the centre of area of that surface, and for the whole of the compartments by

$$\Sigma I,$$

where I denotes the moment of inertia for any one compartment.

The reduction of effective metacentric height will be $\frac{\Sigma I}{V}$, where V denotes the volume of displacement.

To complete the data from which to determine the stability the metacentric height is calculated for the dock and system at each of several selected draughts, and curves are plotted thereto.

To obtain this information accurately it would be necessary to have before us the displacement diagram—which usually contains, amongst others, the curves of displacement, centre of buoyancy, and height of metacentre—relating to the vessel to be docked or for which the dock is to be designed. The moment of inertia of the water plane of the vessel at different draughts is also required, and may be obtained, without constructing a separate curve, from the curves of displacement and height of metacentre, since the moment of inertia of water plane area = height of metacentre above centre of buoyancy × volume of displacement.

When a vessel is being docked such information as is given on the displacement diagram is not generally available, and, moreover, the vessel, although stable, may only possess a small amount of initial stability. It is therefore necessary to allow for such a contingency, in the design, by assuming the vessel to have *no* metacentric height, *i.e.*, that the centre of gravity coincides with the metacentre, in which case we obtain the maximum height of centre of gravity of the vessel when about to take the blocks; the dock is then endowed with sufficient metacentric height as to render the *system* stable.

It is evident that by assuming the metacentric height of the vessel to be *nil* the actual stability of the system will be greater than that estimated.

It may happen that a steamer is initially unstable just prior to docking and rests in equilibrium in an inclined position ; in this case she would be brought into the upright position by ballasting before being placed on the blocks in order to ensure

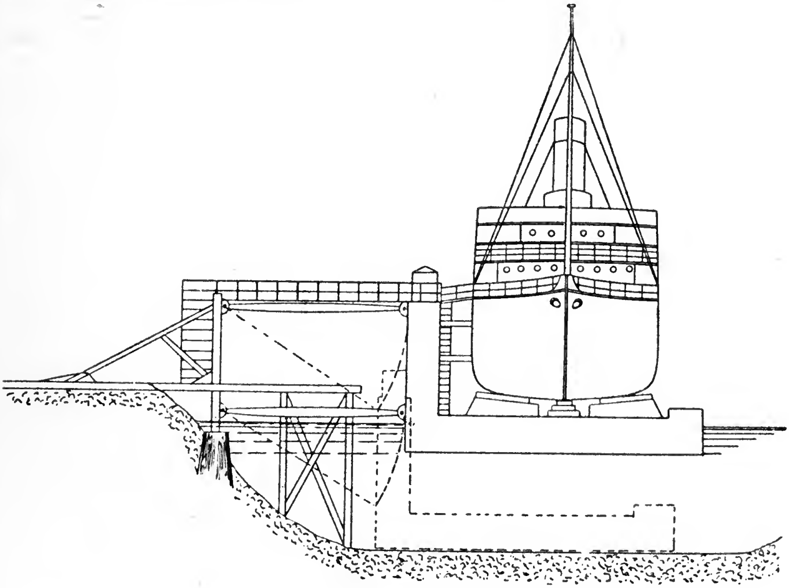


FIG. 81.

safety, and distribute the pressure taken by the shore and bilge blocks equally on both sides of the vessel.

In the event of no exact information being obtainable concerning the displacement, metacentre, etc., of the vessel, such particulars could be estimated approximately by coefficient, as explained in Chapter II.

In the cases of the two types of L dock to be considered, viz., the offshore dock and the outrigger dock, the dock proper is attached in the one case by two parallel tiers of booms to land ties as indicated in Fig. 81 ; and in the other to uprights extending above and below a separate floating pontoon as indicated in Fig. 86. The booms work on centres or pins at their extremities, thus giving the dock freedom of action as

it rises or falls with change of draught or with variation of the tide.

In the case of the offshore dock, however, the inner end of the upper tier boom is pinned to a cam* working on a centre, the cam being so weighted that a limited freedom of motion of the boom end is permitted and with it the dock itself; the maximum displacement or movement of the boom end occurs when the cam is pulled out straight, in which case there is a direct pull on the land ties in consequence of the force acting along the boom.

Without considering for the present the action of the booms, or the booms and outrigger pontoon in the case of the L docks, and assuming the absence of free water in the compartments, we have generally for the system, *i.e.*, the dock and vessel combined :—

$$\text{Metacentric height} = \frac{I_s - V_s \, . \, D}{V_s},$$

where : I_s = moment of inertia of water plane area relating to the system.

V_s = volume of displacement of the system.

D = vertical distance between centres of buoyancy and gravity for the upright position.

For a *small* angle of inclination θ

The "Righting Arm" is given by $\dfrac{I_s - V_s \, . \, D}{V_s} \, . \, \theta$,

and the "Statical Stability" by $(I_s - V_s \, . \, D) \, w \cdot \theta$,

where w is the weight per unit volume of water.

Any free water in the compartments will have a negative moment represented by

$$w \, . \, \theta \Sigma I$$

where I = moment of inertia of free surface of water in any one compartment.

The resultant righting arm now is

$$\left(\frac{I_s - \Sigma I}{V_s} - D\right) \theta,$$

and the resultant statical stability

$$(I_s - \Sigma I - V_s \, . \, D) \, w \, . \, \theta. \quad . \quad . \quad (a).$$

* See Figs. 93 and 94.

If the dock only be considered then I_s and V_s must be replaced by I_d and V_d respectively, where the latter symbols refer to the dock only, and D would naturally be modified accordingly.

For the system to possess positive initial stability we must have the condition

$$I_s - \Sigma I - V_s \,.\, D \quad \ldots. \text{ is positive.}$$

If this expression gives a negative value the system is unstable. Such a condition in the case of the double-wall dock would not be permitted to exist. In the case of the L dock, however, any instability would at once be counteracted by the action of the booms in the offshore dock, and by the joint action of the booms and outrigger pontoon in the outrigger dock, of which the pontoon, although possessing no lifting power, supplies to the system the necessary righting moment for stability.

It will therefore be readily realised that the booms play a very important part in establishing the safety of the system. It must not, however, be imagined that the action of the booms is to forcibly keep the dock in its desired position; the dock and ship should be almost absolutely balanced by the disposition of the water ballast, the booms being chiefly used to indicate the direction in which the dock tended to move out of level and to balance it by the smallest of forces acting along them. Should, however, any extraneous forces act upon the dock so that the balance is upset, the booms then would supply the effort necessary to restore the balance until, should such extraneous action be prolonged, the disturbance is corrected by a readjustment of the water ballast.

Fig. 82 is a diagrammatic representation of an offshore dock floating in such a position of equilibrium that H (centre of buoyancy) and G (centre of gravity) are as indicated. In the position shown there is a tendency to rotation in anti-clockwise direction, which is measured by the couple $W \times z$. If F and P be the forces called into play in the upper and lower tiers of booms respectively, it is clear that for equilibrium $F = P$, since the booms are parallel.

The greater the value of the product $W \times z$, or the greater the value of z for a given value of W, the greater will be that of F and P, since the couple formed by F and P is equal to $W \times z$.

Obviously then, in order to reduce the straining action on the

booms to a minimum, z should be small; this may be brought about by so disposing the water ballast as to bring H and G into, or practically into, the same vertical (longitudinal) plane, or perhaps with G situated a little nearer the wall of the dock than H, so as to obtain a couple (W $\times$ z) tending to give the dock a slight tilt backward, at the same time supplying an element of safety in docking by causing the vessel to lie inwards. The freedom of action of the upper tier boom by virtue of the cam situated at the inner end f_1 allows of this being done.

By increasing the immersion or draught the centre of buoyancy H moves inwards toward the wall of the dock, immediately after the pontoon deck or the top of the air box B

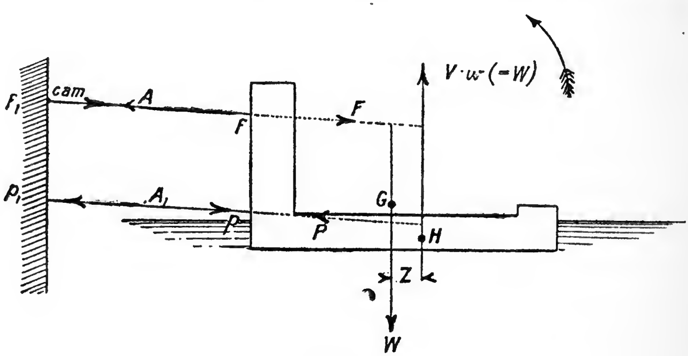


FIG. 82.

becomes submerged, and the disposition of the ballast may be such that a couple W $\times$ z tending to produce angular displacement in clockwise direction results as indicated in Fig. 83, when the nature of the stresses in the booms will be reversed, so that A now experiences tension and A_1 compression.

The booms being pin jointed at the ends will experience no direct bending action, but generally will be subject to compressive or tensile stresses in the manner stated above; and upon the allowed maximum value of these stresses will the design of the booms depend.

If k be the vertical distance ($= fp$ or f_1p_1) between the tiers, ψ the angle which the booms at any time make with the horizontal and n the number of booms in either tier, then

$$F = \frac{W \times z}{n \cdot k \cos \psi}$$

is an equation giving the straining action on the booms. Suppose now that a force R—say due to wind pressure—were introduced, thereby disturbing the equilibrium of the system. If R be more than of a transient nature the system will settle

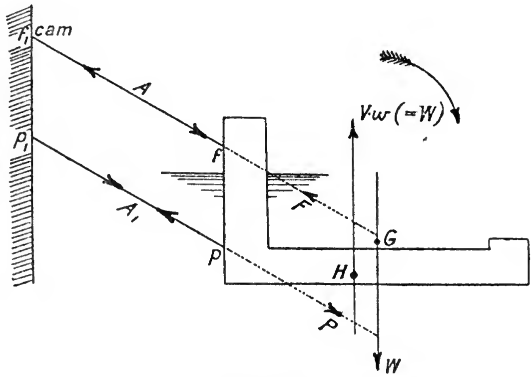


Fig. 83.

down—so long as R continues to act—into a new state of equilibrium in which the following conditions will be experienced (see Fig. 84).

F and P will not now be of equal value, nor will $V \times w$

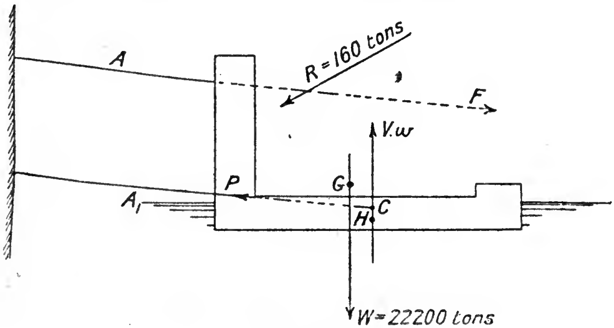


Fig. 84.

(weight of water displaced) be equal to W (weight of the dock or system). If c be the point of intersection of the lines of action of P and $V \times w$, and r, p and m be the perpendicular distances of c from the lines of action of R,F and W respectively, then for equilibrium we have

$$F = \frac{R \cdot r + W \cdot m}{p};$$

the other forces, viz., P and V $\times$ w, may be similarly obtained, but it is more instructive to draw the funicular polygon of forces as shown in Fig. 85, in which the values of R and W are assumed to be 160 tons and 22,200 tons respectively. In this way the values of P and V $\times$ w under the new conditions may be obtained. From the above F = 430 tons, where r = 30 ft., p = 24 ft., and m = 25 ft.

Taking now the case of the outrigger dock, the necessary righting moment is supplied by the outrigger pontoon, which, by the aid of the booms A and A_1 (see Fig. 86), keeps the dock in equilibrium. The pontoon ab generally floats immersed to about its half depth, and its position is invariably one of perpendicularity to the dock wall, since the rectilinear figure formed by the booms ff_1, pp_1, the dock wall fp, and the upright f_1p_1 is a parallelogram, the booms working on fixed centres at f, f_1, p, and p_1.

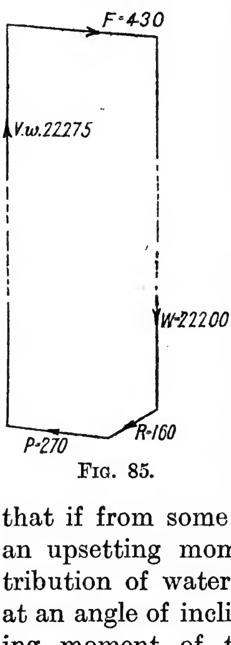


Fig. 85.

The centre of gravity of the pontoon and its equipment is generally situated at a position not far removed from that of the centre of buoyancy, and therefore considerably below the metacentre, in consequence of which the pontoon will possess a righting moment when subject to angular displacement—at least for a considerable angle of inclination. It is clear, therefore, that if from some cause the dock or system were subject to an upsetting moment, due, it may be, to an improper distribution of water ballast in the dock, it would come to rest at an angle of inclination from the upright such that the righting moment of the pontoon just balances the upsetting moment of the dock.

Further, suppose the dock were subject to the action of some extraneous force so that its equilibrium is disturbed and the angle of inclination altered, then, in order that the new position may be one of stable equilibrium, it is necessary, whatever may be the angle of inclination reached, that the heeling moment of the extraneous force is equal to the righting moment of the dock and pontoon considered jointly, under which

condition it is clear that the dock may or may not possess a righting moment—dependent upon the direction of inclination.

It follows, therefore, since the pontoon always possesses a righting moment, that the moment of the dock must be equal but of opposite sign to the algebraic sum of the moments of the pontoon and the extraneous force.

Generally it may be stated that, for equilibrium, the righting moment of the pontoon must balance the moment of the dock acting in the opposite angular direction; and for any other

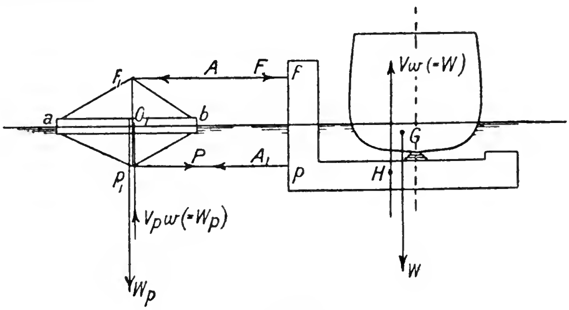


FIG. 86.

position in which equilibrium does not exist the algebraic sum of the moments of the pontoon and the dock must be positive, assuming that the moment of the dock is always positive.

The initial stability of the pontoon is given by $\frac{I_p - \Sigma I_1}{V_p} - D_p$, where I_p is the moment of inertia of the pontoon water plane about a longitudinal axis through o_1.

Assuming the centres of buoyancy and gravity to coincide and that there is an absence of free water in the pontoon, then ΣI_1 and D_p both become zero; the above expression reduces to $\frac{I_p}{V_p}$, and the statical stability of the pontoon for a small angle of heel θ would be

$$I_p \,.\, w \,.\, \theta = \frac{I_p}{36} \,.\, \theta \quad . \quad . \quad . \quad . \quad . \quad . \quad (b)$$

when referred to fresh water, where $w = \frac{1}{36}$ ton.

And for stable equilibrium the *algebraic* sum of the expressions (a) (given on p. 188) and (b), viz. :—

$$\{I_p + (I_s - \Sigma I - V_s \,.\, D)\}\; w \,.\, \theta \text{ must be positive.}$$

This will be better understood by indicating graphically on the same diagram, and to the same scale, curves representing separately

$$I_p \cdot w \cdot \theta,\ (I_s - \Sigma I - V_s \cdot D)\, w \cdot \theta, \text{ and } (I_d - \Sigma I - V_d \cdot D)\, w \cdot \theta$$

the stability of the pontoon, the system, *i.e.*, dock and vessel combined, and the dock only respectively for an angle, say, of 1° inclination.

Such a diagram is shown in Fig. 87, and relates to the pontoon of dimensions 320 ft. × 70 ft. × 5 ft. by 2·5 ft. draught,

Curves—D and S refer to the dock, and P to the outrigger pontoon—of "statical stability" for 1° inclination, and of "initial stability."

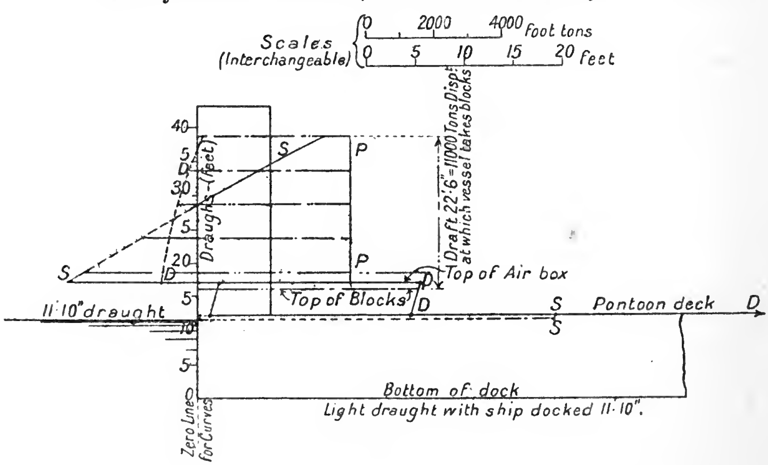


FIG. 87.

designed to work the dock outlined in Fig. 88, this latter being required to lift a vessel of about 11,000 tons displacement.

The curve "P" relating to the pontoon is of course a straight line parallel to the line of ordinates (draughts); and the curves "S" and "D" refer to the system (dock and vessel) and the dock only respectively.

On examining these curves it will be noticed that the stability both for the system and the dock only is *negative* for a portion of the time during which immersion (or emergence) takes place.

For the system this period lies between the draughts 28 ft. and 17 ft.

For the dock only this period lies between the draughts 36 ft. and 17 ft.

If, however, we combine P with S and P with D it will be seen that there is always existent positive stability for the *entire system, i.e.*, for the pontoon and dock combined, and for the pontoon, dock, and ship combined.

To illustrate the application of the foregoing principles, an example is worked out below relating to an offshore dock, designed to lift a passenger steamer of dimensions 490 ft. × 56·5 ft. × 37·5 ft. moulded depth, the displacement of the vessel being 10,700 tons on 22 ft. 3 in. mean draught.

The general outline and dimensions of the dock are indicated in Fig. 88, while the curves relating to the vessel and used for the purpose of calculating the stability of the system are shown in Fig. 89.

Fig. 90 shows by means of the curve S the *actual path* or locus of the centre of buoyancy of the system, while curve D refers to the dock only; to the left of the figure are drawn two curves S_1 and D_1, giving respectively the height, above the bottom of dock, of the centres of buoyancy of the system and dock. By combining the two sets of curves we are able to

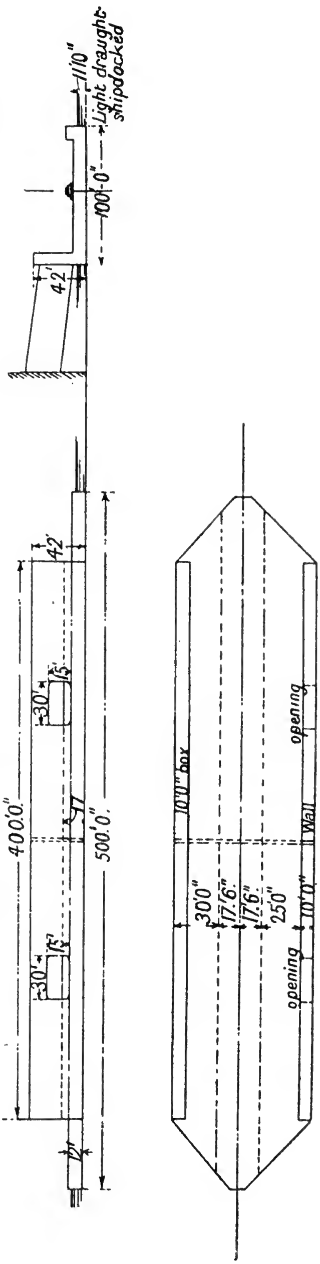


Fig. 88.

determine the exact position of the centre of buoyancy relative to the vertical longitudinal plane through the centre of block.

For example, suppose we required the actual position of the centre of buoyancy for the dock only, at 25 ft. draught. From the curve D_1 measure the height of centre of buoyancy above the bottom of dock at the 25 ft. draught mark, and draw

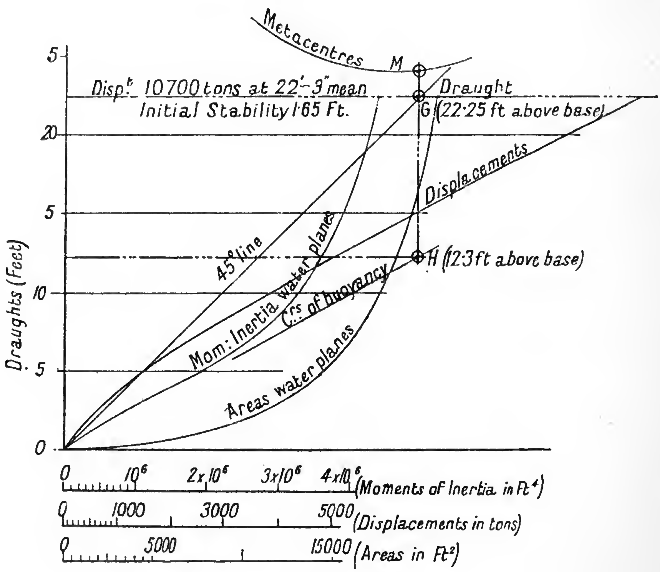


FIG. 89.

a line parallel to the dock bottom at this height above it to cut the curve D; the point of intersection is the position of the centre of buoyancy required. If this point be now joined to that representing the centre of buoyancy of the ship corresponding to this draught (*i.e.*, V_9 at a draught of 9 ft. for the ship) and the joining line be divided inversely as the displacements of the dock and ship, the point so obtained will be the centre of buoyancy of the system for a draught of 25 ft. This is indicated by S_{25} in Fig. 90. By obtaining a number of such points and drawing a fair curve (S) through them the locus of "centre of buoyancy" for the system is obtained.

To obtain the position of the centre of gravity of the system at a draught say of 28 ft. 3 in. we may proceed as follows :—

Displacement of dock .	17,150 tons	
,, ,, ship .	5,100 tons (at draught for ship of 12 ft. 3 in.).	
Total displacement .	22,250 tons . .	22,250 tons.
Estimated weight of dock.	4,100 tons	
,, ,, ship.	10,700 tons.	
Weight of dock and ship .	14,800 tons . .	14,800 tons
,, ballast contained in the dock . .		7,450 tons.

Diagram showing
Curve S—locus of centre of buoyancy of System.
,, D— ,, ,, ,, Dock.
,, S_1—height ,, ,, System above base.
,, D_1— ,, Dock ,, ,,

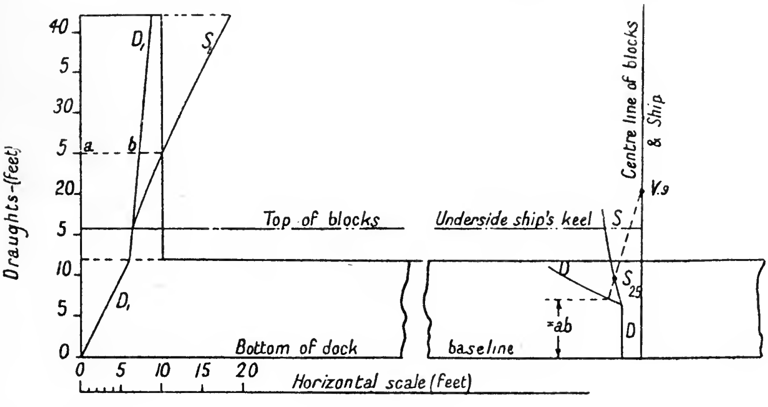


Fig. 90.

The calculated and experimentally obtained positions of the centre of gravity of the dock (light condition) and of the ship are shown on the diagram (Fig. 91), and from these that of their common centre of gravity is easily determined, viz., G_1.

Assuming that the centre of gravity of the system is to be situated in the same vertical longitudinal plane with the centre of buoyancy, the position of which latter may be read off from Fig. 90, we are able to determine the exact position

which the centre of gravity of the ballast must occupy as follows :—

Referring to Fig. 91.

(*a*) $\frac{14{,}800\,(4{\cdot}84 - 3{\cdot}85)}{7{,}450} = 1{\cdot}97$ ft. = distance of centre of gravity of the ballast from the vertical longitudinal plane through the centre of buoyancy H ; *this is the condition that the centres of gravity and buoyancy of the system shall be in the same vertical longitudinal plane.*

(*b*) The vertical position of the centre of gravity of the ballast and that of the centre of gravity of the system may be determined by calculation as follows :—

The ballast is first disposed transversely in such a manner that the position of its centre of gravity is situated horizontally 1·97 ft. from that of the centre of buoyancy of the system ; this is indicated by G_2 in the Fig. 91 and its vertical position together with that of the system is estimated thus :—

Item.	Weight (tons).	Levers * (feet).	Moments.
Ballast . .	620	2·8	1,736
	1,830, 1,425, 1,425	3·0	14,040
	2,150	3·05	6,557
	7,450	3·0	22,333
Ship and dock.	14,800	31·25	462,575
Whole system	22,250	21·8	484,908

i.e., with the centre of gravity of the ballast situated 3·0 ft. above bottom of dock, the centre of gravity of the system is situated 21·8 ft. above the same level.

Therefore HG = (21·8 — 11·12) = 10·68 ft.

At the above draught calculating the values of I_s, ΣI and V_s we obtain $\frac{I_s - \Sigma I}{V_s} - HG = +\cdot 8$ ft. (nearly).

For the dock only the expression gives — 1·0 ft. (nearly).

These are the values of the initial stability of the "system" and the "dock" only, respectively at 28 ft. 3 in. draught.

The disposition of the water ballast indicated in Fig. 91 is not necessarily what would be actually obtained in practice,

* Levers taken about bottom of dock.

but is such that the initial stability resulting therefrom is about as great as it possibly can be, for the given draught. The

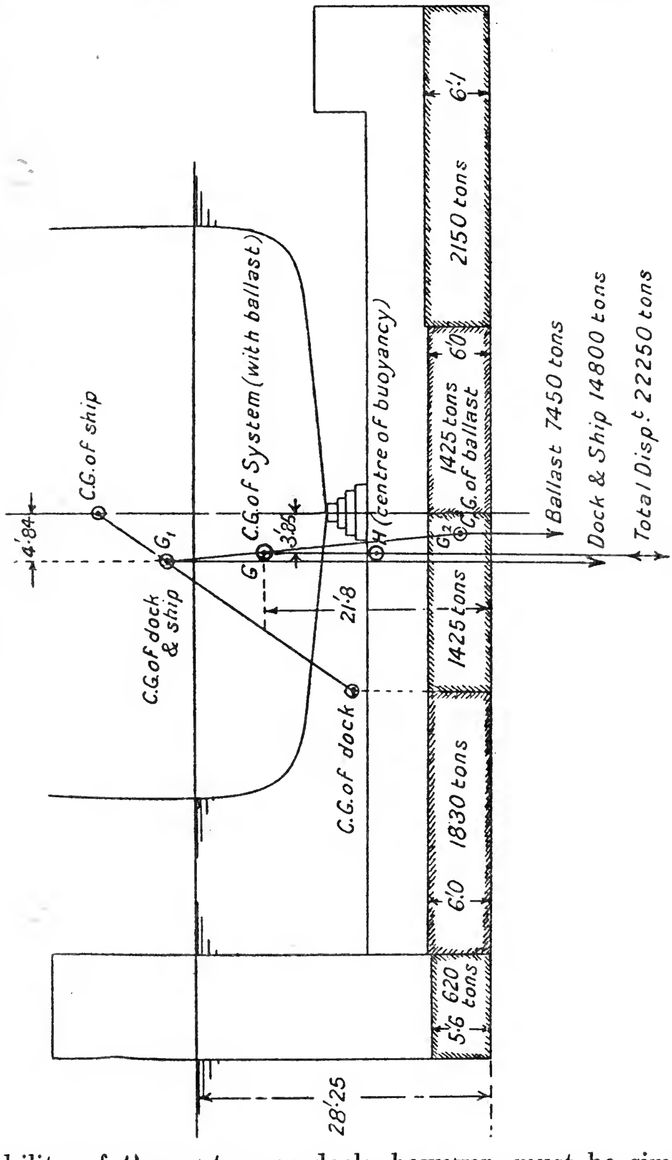


FIG. 91.

stability of the system or dock, however, must be similarly examined for other draughts ranging from the time at which

the ship first touches the keel blocks until she is completely lifted and the pontoons emptied, and from the results obtained we are able to construct a diagram showing graphically the variation of initial stability with draught.

The curves S and D in the diagram (Fig. 87) drawn to represent the statical stability for 1° inclination may be used also to represent the initial stability by altering the scale so that the horizontal ordinates of the curves represent feet.

This scale is shown in the diagram.

The scales are therefore related, inasmuch as any ordinate of the curve when read off for initial stability and multiplied by ·0175 and by the displacement of the vessel gives the statical stability at the particular draught chosen.

Consider the system floating with the top of the air box just immersed, *i.e.*, in that condition in which the maximum negative stability is reached. We will determine the stress in the booms.

The initial stability of the system = — 12·8 ft.

The statical stability of the system for one degree of inclination = — 12·8 × 16,150 × ·0175 = — 3,617·6 foot-tons.

Let P be the force acting along any one boom.

ψ denote the inclination of the boom to the horizontal.

$n = 6 =$ the number of booms in each tier.

$k = 25$ ft. = the vertical distance between the tiers.

$$\text{Then } P = \frac{3617{\cdot}6}{6 \times 25 \cos\psi} = \frac{24{\cdot}117}{\cos\psi} \text{ tons.}$$

It is evident that the force P acting along the booms will be a maximum when the ratio of " Statical Stability " to " cos ψ " is a maximum, the statical stability being *negative*. For suppose the dock (or system) at rest under normal conditions in a position of stable equilibrium ; the lines of action of the weight and buoyancy are coincident and therefore the force P is zero ; if the dock were displaced from that position it would when free return to the initial position without the aid of the booms. But suppose while at rest in the normal condition that the dock possess negative stability due to the non-coincidence of the lines of action of the weight and buoyancy, it may be kept in that position only by the effort of the booms, the moment of the forces acting along them balancing the

negative couple of the dock and therefore the moment of the forces in the booms, and the forces themselves will be a maximum when, for any particular angle of inclination ψ of the booms, the *negative* stability of the dock is greatest. The angle ψ will depend upon the length of the booms and the positions of the points of attachment of the boom ends relative, say, to the bottom of the dock, and will vary with the draught. In the present case the outer end of the lower tier of booms is attached at the level of the top of the pontoon; the booms are horizontal when the dock is floating at a draught of 11 ft. 10 in. and are 50 ft. long.

At the draught considered, viz., that at which the top of the air box is just awash, $\cos \psi = \cdot 99$,

$$\text{and } P = \frac{24{\cdot}117}{\cdot 99} = 24{\cdot}36 \text{ tons.}$$

A curve SS showing the variation of the above ratio of "statical stability" to "$\cos \psi$" with draught is drawn in Fig. 92; the broken portion of the curve applies to those draughts at which the system possesses positive stability. A curve DD, referring to the dock only, is also drawn on the same diagram.

It will be seen that the maximum *negative ratio* occurs at the draught 17 ft., its value being 3,655.

The value of P for this condition is $P = \dfrac{3{,}655}{6 \times 25} = 24{\cdot}36$ tons (as above), and if we assume a factor of safety of 4 and a maximum angle of heel of $1\frac{1}{2}°$ for purposes of design we have :—

$$\text{Strength of boom} = 24{\cdot}36 \times 4 \times 1\tfrac{1}{2} = 146{\cdot}16 \text{ tons.}$$

In some cases, in order to obtain a sufficient margin of strength, dock designers endow the dock with a metacentric height in excess of the actual amount, the amount selected depending very largely upon the speed with which the dock has to be worked. Such a margin of strength, however, may be included in the factor of safety by assigning to the latter a large enough value.

Suppose now that the dock instead of being of the off-shore type were required to be worked with an outrigger pontoon.

Taking the pontoons of the dimensions given on p. 194, viz. :—

320 ft. × 70 ft. × 5 ft. by 2·5 ft. draught,

we have :—

$$I_p = \frac{320 \times 70^3}{12} = 9,146,666 \text{ (ft.)}^4$$

$$V_p = 320 \times 70 \times 2{\cdot}5 = 56,000 \text{ (ft.)}^3.$$

The centres of gravity and buoyancy of these pontoons are

CURVES SHOWING VARIATION OF THE RATIO OF "STATICAL STABILITY" TO COS ψ.

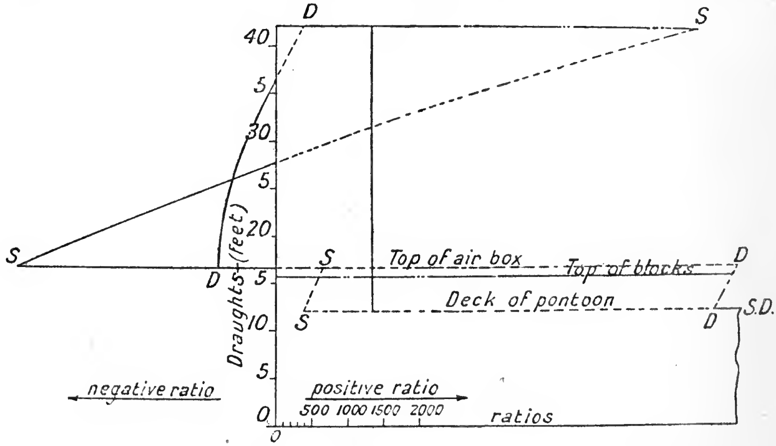


FIG. 92.

practically coincident; therefore the initial stability of the pontoon $= \dfrac{9,146,666}{56,000} = 163{\cdot}3$ ft., and the statical stability of the pontoon for a small inclination θ is :—

$$163{\cdot}3 \times \frac{56,000}{36} \times \theta = 254,074\ \theta \text{ foot-tons} \qquad . \quad . \quad (a_1)$$

Now it was seen that the statical stability of the dock and vessel for that condition in which the top of the air box is just immersed is :—

$$-\,206,720\ \theta \text{ foot-tons} \qquad . \quad . \quad . \quad (b_1)$$

Therefore the statical stability of the pontoon, dock, and vessel taken collectively is $a_1 + b_1 = 47,354\ \theta$ foot-tons.

Should the value of $a_1 + b_1$ at any time be negative—*i.e.*, should the *upsetting* moment of the dock, or the dock and vessel,

exceed the righting moment of the pontoon—disaster would probably result.

$a_1 + b_1$ then represents the tendency to restore the whole system to the initial position of equilibrium when held forcibly or momentarily at a given inclination; when free that position will ultimately be obtained, and may be such that

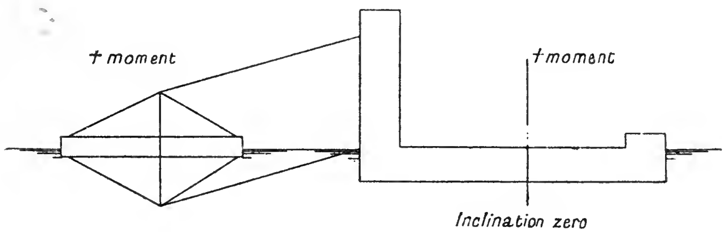


FIG. 92A.

either both the pontoon and the dock assume the upright (Fig. 92A), or that they come to rest inclined at such an angle that the negative moment of the dock is equal to the positive moment of the pontoon, in which the inclination of the dock and the pontoon are in the same direction (Figs. 92B and 92C).

The moment of the pontoon or of the dock when equilibrium

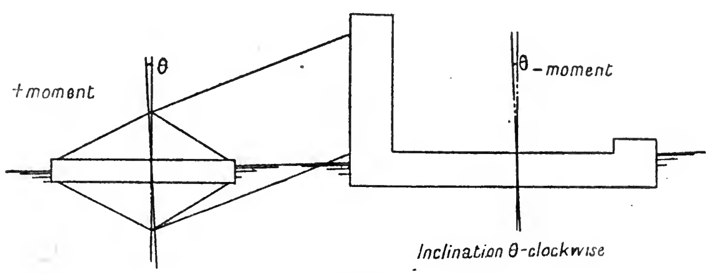


FIG. 92B.

is established is also the moment of the forces acting along the booms.

It will be evident, whatever may be the type of dock, that its stability will be dependent upon the distribution of the water ballast, and therefore will be almost entirely in the hands of the dockmaster or the official under him actuating the valves, much in the same way that the stability of a ship is

dependent very largely upon the manner in which the stevedore stows the cargo in loading the vessel for a voyage.

The distribution of water ballast outlined in the above example is given to illustrate the application of mechanical principles in determining the amount, and position of the centre of gravity, of the ballast contained in the dock at any particular draught necessary to produce the requisite amount of stability. Such calculations are of service as a guide in pumping water into or out of the compartments, according as the dock is required to sink or rise, and to indicate not only the amount of water, but also the compartments to be dealt with at any specific stage of the operation of docking. In actual practice such knowledge asserts itself almost unconsciously, and by long experience the operator comes to know by the behaviour

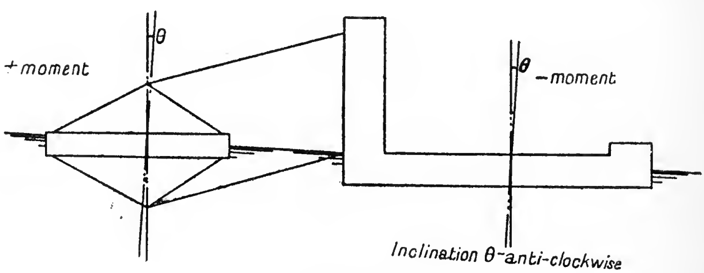


Fig. 92c.

of the dock itself which compartments should be dealt with and to what extent they are to be filled with or depleted of water ballast.

It was stated when dealing with the offshore dock that the inner end of the upper boom was attached to a weighted cam by which the dock, while kept in a state of stability, was allowed some limited freedom of motion laterally ; the action of the cam will now be explained.

Fig. 93 shows the general arrangement of the cams—two of which are attached to the boom—and the way in which they are connected to the uprights and the boom.

The cams work about a centre o (Fig. 94), the centre of attachment o_1 of the boom to the cams being 6 in. from it. It is seen that when oo_1 is vertical the force acting along the boom is *nil*. Suppose the dock deflected through a small

angle θ, it is evident that the deflection of the cam illustrated in Fig. 94 will be considerably greater. If, for example, the dock be floating in such a position that the booms are horizontal, then the horizontal distance moved through by the upper boom, and therefore by the point o_1, is $k \cdot \theta$, represented by $o_1 n$ in the figure, where k is the vertical distance between the tiers of booms.

The deflection of the cam is therefore $\sin^{-1} \frac{o_1 n}{oo_1} = \sin^{-1} 50 . \theta$, $= \varphi$, where $k = 25$ ft. and θ is the circular measure of the angle of inclination of the dock.

If equilibrium be now established in the new position and P

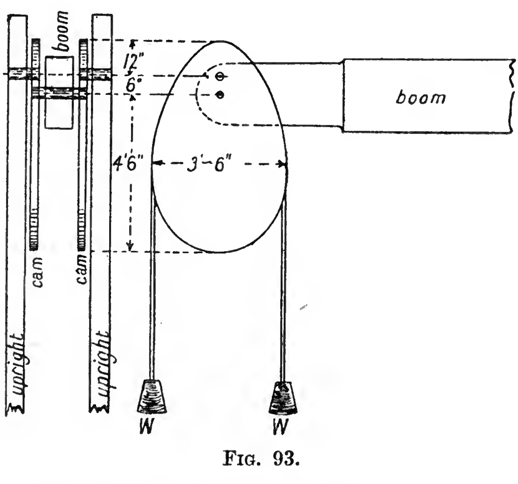


FIG. 93.

the force acting along the boom, we have :—

$$P \cdot on = W(ab - bc) = P \cdot oo_1 . \cos \varphi.$$

It will be seen that for only a moderate deflection of the cam the value of $(ab - bc)$ becomes considerable, and when the cam is pulled out straight would be 4 ft.

Suppose the dock deflected through 1°, then :—

$$\sin \varphi = 50 \times \cdot 0175 = \cdot 875$$

$$\therefore \cos \varphi = \cdot 484,$$

and $(ab - bc)$ for this position $= 2$ ft. 9 in.

Hence $P \times \cdot 5 \times \cdot 484 = W \times 2 \cdot 75$

$$\text{or } P = 11 \cdot 36 \text{ W}.$$

If we take W = 1 ton, then P = 11·36 tons.

The weight W may be suspended from a looped cord, the cords of the loop passing over and fixed to the two cams so that the pull on each cam is $\frac{W}{2}$. The weights may also be so arranged as to rest on a platform placed at the necessary height when the cams (*i.e.*, oo_1) are vertical. Under such a condition there is no pull on the cams, but when the latter become deflected the weight W on one side remains at rest while the other only is acting, in which case the moment about o is $W \cdot ab$ instead of $W(ab - bc)$, as before.

It will be seen that the outrigger pontoon performs a function similar to that of the cam described above. In this case, while the dock acquires a certain amount of angular movement, the restraining force acting along the booms is brought into being by the action of two equal forces, viz., the buoyancy and weight of the outrigger pontoon, much in the same way as the force acting along the booms in the case of the offshore dock finds existence by virtue of the weight W acting on the cam.

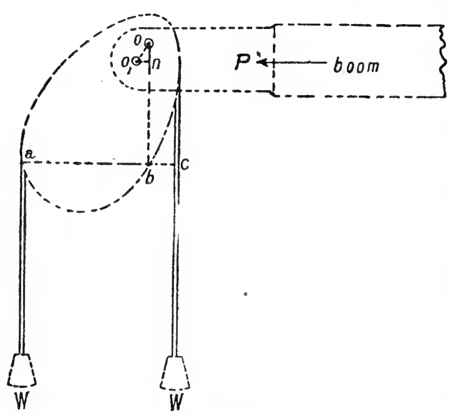


Fig. 94.

The strength of the boom in the case of the outrigger dock is determined in the same way as for the offshore dock, the moments of the forces acting along the booms at any time when the dock is in equilibrium at rest being equal to that of the righting moment of the pontoon or the upsetting moment of the dock.

It has been explained that the double-wall dock presents the simplest problem, for the reason that the structure is symmetrical with relation to the longitudinal vertical plane passing through the middle line of the blocks; this symmetry is still retained after the vessel has been centred for docking.

The formula for the value of statical stability when the system is inclined through a small angle θ is given on p. 188, viz. :—

$$(I_s - \Sigma I - V_s \,.\, D)\, w \,.\, \theta \text{ foot-tons};$$

and the condition for stability is that

$$I_s - \Sigma I - V_s \,.\, D \text{ is positive.}$$

For the dock only this expression becomes

$$I_d - \Sigma I - V_d \,.\, D,$$

where the symbols contained in the expression now refer to the dock only.

The diagram (Fig. 95) gives the height of metacentre above the bottom of the dock in the case of the double-wall dock of dimensions indicated in Fig. 96. Curve D refers to the dock only and curve S to the system (dock and vessel). The curve V, referring to the vessel only, shows the height of metacentres above the bottom of keel or the top of blocks and relates to the same vessel as was considered when dealing with the L dock ; this curve is included principally to illustrate the difference in character between it and those relating to the dock and the system. The irregularities of the latter are due to the following causes :—Openings in the dock walls, variation in the horizontal section of the dock walls as the pontoon deck is approached, and the sudden change of horizontal section or water plane area from that of the dock walls to that of the walls and pontoon together for draughts not exceeding the depth of the pontoon.

On analysing the diagram, the effect of the vessel in modifying the *dock* curve D and producing the resultant curve S will be clearly seen. At *o*—at the level of the top of blocks—the curve for the system joins that relating to the dock only, and for draughts less than 16 ft. the two curves are identical.

The period from *o* to *p* during which the dock emerges from the draught of 16ft. to that of 12 ft.—at which latter the pontoon deck becomes awash—has been termed the *critical* period. From the point *p* the curve suddenly breaks away horizontally to *q*, due to the sudden increase in the moment of inertia of the water plane area, from that of the section of the dock walls to that of the combined section of the walls and the pontoon ; from the point *q* the curve rises to its maximum value at the light draught of the dock.

The question of longitudinal stability would be treated in a manner similar to that in the foregoing consideration of transverse stability ; and it is therefore unnecessary to deal with the matter here, beyond stating that the walls must be of such a length and breadth as to ensure absence from tipping longitudinally under ordinary circumstances.

It will be apparent from the foregoing remarks that the motion of the floating dock during an angular displacement is generally

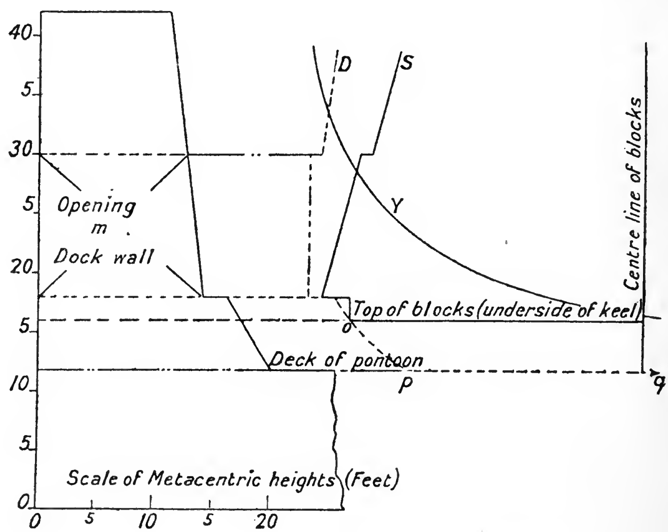


Fig. 95.

Curve D—height of metacentres above dock bottom (for dock only).
,, S— ,, ,, ,, ,, (for system—dock and vessel).
,, V— ,, ,, ,, underside of keel (for vessel).

quite different from that of an ordinary vessel. In the latter case the vessel is absolutely free to move, while the motion of the dock is more or less constrained, due to the action of the booms in the case of the offshore dock and of the outrigger pontoon and booms in that of the outrigger or depositing dock.

The subject of the centre or axis of rotation will therefore be found to be both interesting and instructive, and will now be considered in the case of the L docks, that of the double-wall type needing no further reference,

since, except for its moorings, its motion would be considered in a manner similar to that of an ordinary vessel,

(*a*) Determination of the instantaneous axis of rotation of the offshore dock (see Fig. 97).

Assuming the volume of displacement to remain constant as the dock heels through a small angle $d\theta$, then the new water line will pass through o, the centre of area of the initial water plane, and with any movement of the dock the motion of the point o will be horizontal along the surface of the water.

The point p—the outer end of the lower boom fp—will move in a circular arc of radius fp and centre f, since f is a fixed point,

If we now consider the two points o and p, since o moves horizontally, the instantaneous centre of rotation will lie in the vertical through o, and since p moves in a circular arc about f, the instantaneous centre lies in fp produced. Therefore c, the point of intersection of the vertical through o with fp produced, is the position of the axis required.

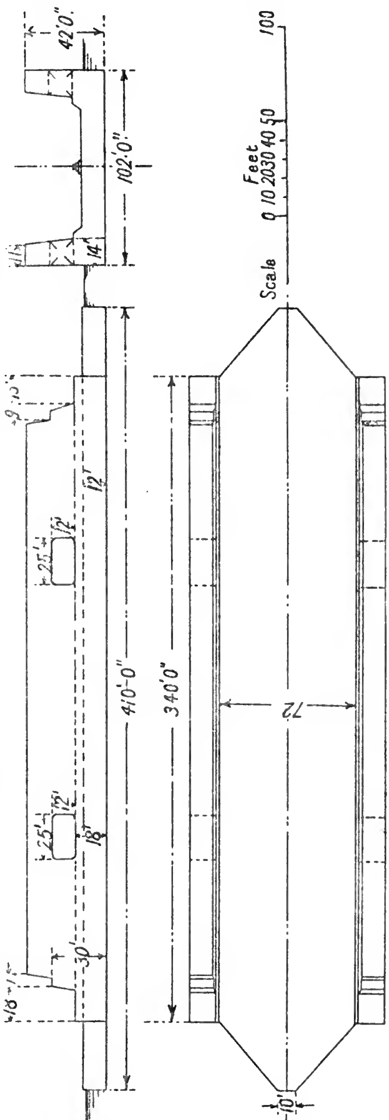


Fig. 96.

The movement or path of the point p_1 cannot be directly determined, since as the dock inclines it will be governed by the movement of the

point of attachment of the upper boom to the cam at f_1. Having determined the centre of rotation c, however, the path of p_1 will be perpendicular to the line cp_1 at p_1.

It will be seen that the position of the instantaneous axis is defined by $cm_1 = b \tan \varphi$, where cm_1 is the vertical distance of c

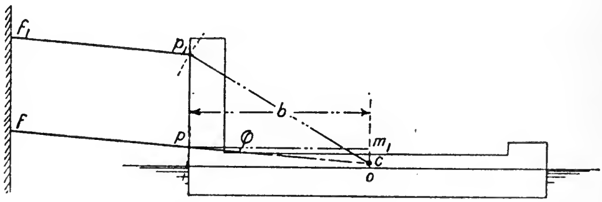


FIG. 97.

from the horizontal line through p, and b the distance of the centre of area of water plane from the outer face of the wall.

φ is the angle of inclination of the boom to the horizontal at the instant considered.

(*b*) Determination of the instantaneous axis of rotation of the outrigger or depositing dock (see Figs. 98 and 99).

Let o and o_1 denote the centres of area of the initial water planes relating to the dock and the pontoon P respectively,

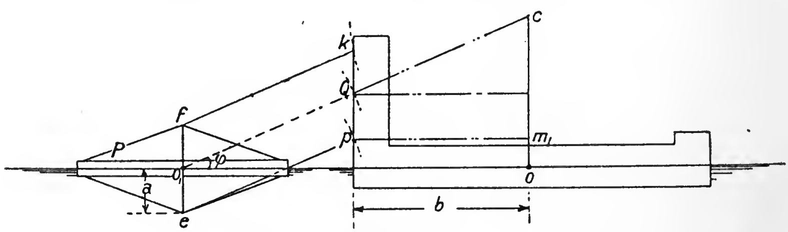


FIG. 98.

then for a small angular displacement $d\theta$ the new water lines will pass through o and o_1.

In considering the relative movement of the dock and the pontoon we may assume o_1 to be fixed ; then any movement of the dock will cause o to move horizontally ; the centre of rotation therefore will be situated in the vertical through o.

First, assume the upright or standard ef to pass through o_1 (Fig. 98), and consider the point Q in the dock wall where o_1 Q is drawn parallel to the booms ep and fk ; then, since the

booms together with ef and pk form a parallelogram, it is easily seen that as the dock moves into a new position the point Q

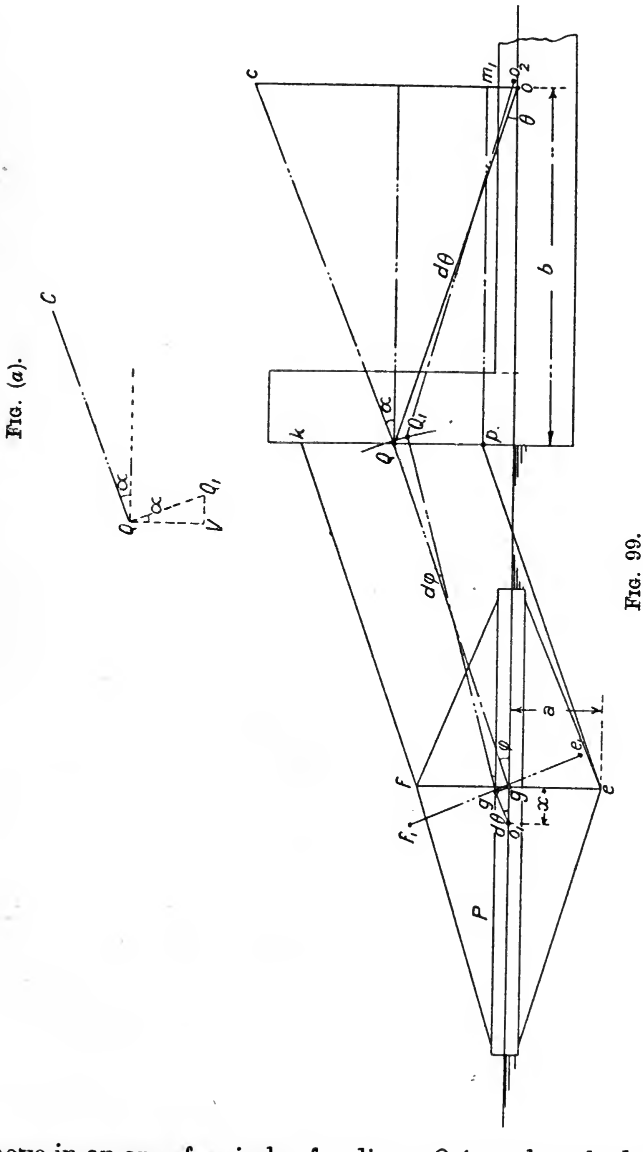


Fig. (a).

Fig. 99.

will move in an arc of a circle of radius o_1Q (equal to the length of the boom) and centre o_1, and therefore the instantaneous

centre lies in o_1Q produced. Therefore c, the point of intersection of o_1Q and oc, is the position of the axis of rotation required, and in such that

$$\begin{aligned} cm_1 &= pQ + pm_1 \tan \varphi \\ &= a + b \tan \varphi, \end{aligned}$$

an expression similar to that obtained in the case of the offshore dock.

It will be noticed that while the paths of e and f are circular arcs described with o_1 as centre, the outer ends p and k of the booms will move in directions perpendicular to cp and ck respectively.

Next, suppose ef not to pass through o_1, but through a point g in the water plane distant x from it (Fig. 99).

Let the dock experience an angular displacement $d\theta$, then, while the pontoon P and with it o_1g suffer the same displacement, the booms will move through an angle $d\varphi$. This is indicated in the figure, where gQ drawn parallel to the booms takes up the second position g_1Q_1 and the line oQ the position o_2Q_1.

Let l denote the length of the booms $= gQ$.

Then the fall of Q vertically is :—

$$l \sin \varphi - l \sin (\varphi - d\varphi) - x \sin d\theta = Qv \text{ in figure } (a).$$

The horizontal movement of Q is :—

$$l \cos (\varphi - d\varphi) + x \cos d\theta - l \cos \varphi - x = Q_1v \text{ in figure } (a).$$

$$\begin{aligned} \text{Therefore } \tan a &= \frac{l\{\cos(\varphi - d\varphi) - \cos\varphi\} + x(\cos d\theta - 1)}{l\{\sin\varphi - \sin(\varphi - d\varphi)\} - x \sin d\theta} \\ &= \frac{l \sin \varphi \sin d\varphi}{l \cos \varphi \sin d\varphi - x \sin d\theta} \\ &= \frac{l \sin \varphi}{l \cos \varphi - x \cdot \dfrac{d\theta}{d\varphi}}. \end{aligned}$$

$$\text{Now } l \sin \varphi = Qo \cdot \sin \theta,$$

$$\therefore\ l \cos \varphi \cdot \frac{d\varphi}{d\theta} = Qo \cdot \cos \theta = b$$

$$\therefore\ \tan a = \frac{b}{b - x} \tan . \varphi.$$

But the instantaneous centre will be situated in Qc, where Qc is drawn perpendicular to QQ_1, the path of Q.

$$\begin{aligned}\text{Therefore } cm_1 &= a + b \tan \alpha \\ &= a + b \tan \varphi \cdot \frac{b}{b - x}\end{aligned}$$

gives the height of the instantaneous centre of rotation above the level of p.

If o_1 and g were coincident so that x is zero, then the above expression becomes

$cm_1 = a + b \tan \varphi$, as in the previous case.

CHAPTER V

AIR CRAFT

THERE are two ways by which a body may be supported in a gaseous fluid, viz., by the buoyant action of the fluid on the body, in a manner analogous to that experienced in the case of a vessel floating at the surface of the water, or by exerting energy in the machine or body itself to create an amount of momentum downwards in the mass of fluid immediately surrounding the body sufficient to counteract the action of gravity on the body and cause it to move upwards.

An example of the first method is found in the ordinary balloon; the second method is adopted in aeroplanes, which, when driven ahead by the contained motor, causes air pressure to be exerted against a plane or planes carried by the machine and inclined to the line or path of flight, such that the vertical component of such pressure acting upwards counterbalances the action of gravity.

Such is a crude description of the means employed to enable balloons on the one hand and aeroplanes on the other to remain, so to speak, suspended in the air.

It will not have escaped notice that the submarine is supported by a combination of two such systems of forces.

An ordinary balloon is very difficult to steer, and so has never proved of any noteworthy practical utility; roughly speaking, it requires some 13·5 cubic ft. of balloon content to support 1 lb.; hence it is easily understood that a useful balloon would require to be of great cubic content, and would experience great resistance to its motion through the air. To reduce this resistance to a minimum the elongated form has been adopted with axis horizontal, and driven by a motor suitably housed or attached to the balloon itself constitutes the "dirigible" as we know it to-day.

The possible success of this type of aircraft was indicated by A. Santos-Dumont, who successfully encircled the Eiffel

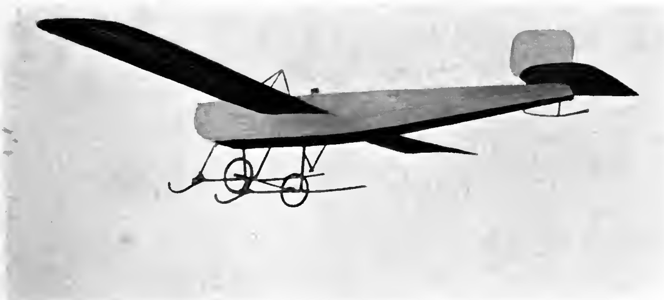


FIG. 100.—" Bristol " Monoplane.

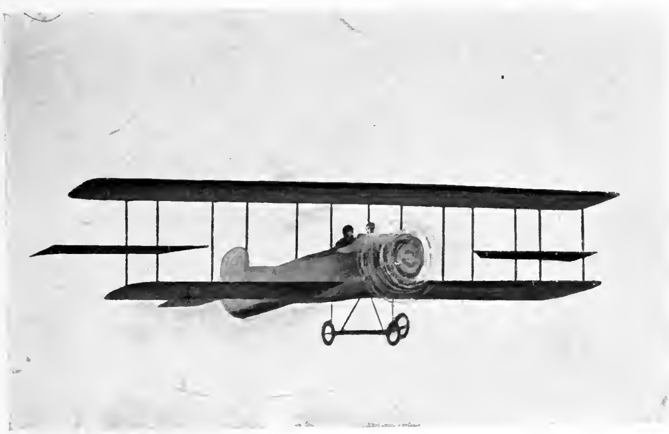


FIG. 101.—Curtiss Biplane.

[*Face p.* 214.

Tower in 1901 in one of his dirigibles at the speed of 18 miles per hour. Count Zeppelin, after repeated and unfortunate disaster to his efforts, made some remarkable passages in the air in dirigibles of his own design. His first airship, 416 ft. long and weighing some 9 tons, was launched in the year 1900, while the last two ships of that name met with disaster, as have their predecessors—one through encountering a gale off Heligoland, the other as the result of explosion while manœuvring in the air in Germany in 1913.

Unlike the balloon, the flying machine may be said to possess no buoyancy, and remains suspended in the air by virtue only of the air pressure acting on superposed planes, as the result of the machine being propelled. There were two early types of this class of machine, viz. :—

(*a*) Ornithopters, which depended upon the reciprocating motion of wings worked mechanically,

(*b*) Helicopters, which were lifted into the air by the action of propellers rotating on a vertical shaft.

Neither of these types of machine has met with success, and the only type now in use is the "aeroplane," a machine having inclined surfaces, which when propelled forward deflect the air, so causing upward pressure sufficient, on a certain speed being attained, to lift it.

The aeroplane may be divided into two classes :—first, the Monoplane ; second, the Biplane.

These machines possess respectively one and two main planes ; the "Triplane" has been experimented with, but so far no advantage has been found to accrue from increasing the surfaces or planes beyond two.

Fig. 100 shows a general view of a monoplane of the "Bristol" class ; while Fig. 101 indicates a "Curtiss" biplane.

It is evident that to lift the machine as it moves forward is of itself insufficient for practical purposes ; the aeroplane must be supported in stable equilibrium—in other words, it must not upset when subject to disturbing influences such as wind gusts ; and, moreover, it should be able to return to its initial position of equilibrium after the disturbing forces have been removed.

In what follows we shall consider the action of the external

or disturbing factors in so far as they may affect the equilibrium of the aeroplane, and the principles governing stability in the longitudinal, and transverse or lateral, vertical directions; and in the horizontal plane when the machine is turning.

The subjects of longitudinal and transverse stability both require careful consideration, and it is not easy to determine which is the more important or which presents the greater difficulty of solution.

All aircraft, whether of the form of the aeroplane or airship, have a tendency to oscillation in the longitudinal direction, due doubtless to the disturbance of the fluid in the immediate vicinity of the body, brought about by the forward movement of the latter, and the rotation of the propeller.

When the aeroplane is in equilibrium in flight, the path

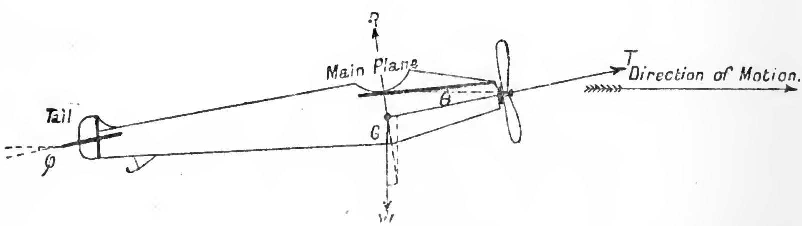


FIG. 102.

pursued—at least momentarily—is straight; the resultant force due to air pressure will be situated in the plane of symmetry, and together with the propelling or tractive force, and the weight of the machine, must form a system in equilibrium.

If the direction of the propelling force T pass through the centre of gravity of the machine, it follows that the resultant air pressure must also pass through this point as shown in Fig. 102. If now the angle of incidence—*i.e.*, the angle at which the air meets the plane or planes, and indicated by θ and φ in the figure—be altered, R, the resultant force due to air pressure, will change in magnitude, position, and direction, the balance of the forces will be disturbed, and the machine will experience angular displacement.

If with this modification the magnitude of T be changed by increasing or diminishing, as the case may be, the speed of

the motor, and the resultant force due to air pressure still pass through G, equilibrium might still be attained, but the machine will pursue a different course or path. This is indicated in Fig. 103.

Such a change of condition may be produced by altering the angle of incidence, and this with a machine flying in still air

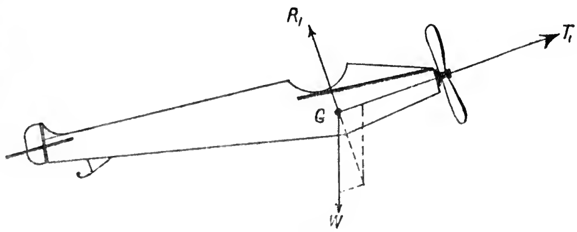


Fig. 103.

can only be brought about by making one or more of the planes to hinge, so that the angle at which the air meets the plane may be controlled mechanically. From this it is evident that an aeroplane fitted with immovable planes can only fly with one particular angle of incidence.

The movable plane therefore forms a very important feature of the design, it is usually situated at the tail of the

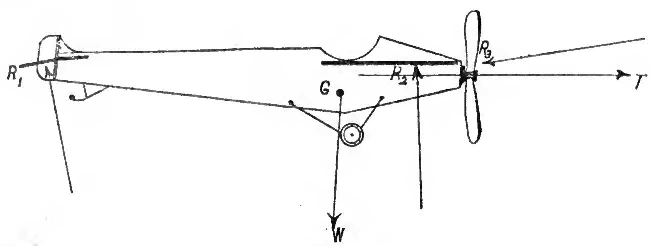


Fig. 104.

machine, and must be so placed relatively to the fixed plane or planes that in order to fly at different angles of incidence the resultant R of the forces due to air pressure must pass through G and balance the resultant of T and W, also passing through this point.

In order to establish equilibrium, however, it is not imperative that the direction of T should pass through the centre of gravity of the machine; indeed, the propelling

force is usually such that it produces a moment relative to an axis through G tending to rotate the aeroplane in the clockwise direction. This is shown in Fig. 104, wherein the direction of T passes above G; let R_1 and R_2 denote the air pressures at right angles to the planes, and R_3 the resistance (other than R_1 and R_2) offered by the air to the motion of the machine.

The conditions will not be altered if we apply at G forces equal in magnitude, and acting in the same and opposite directions to, T, R_1, R_2, and R_3, when the system of forces may be resolved into :—

(*a*) A couple, the value of which is given by :—

$$T \times t + R_1 \times r_1 + R_2 \times r_2 + R_3 \times r_3 = M,$$

where, t, r_1, r_2, and r_3 denote the perpendicular distances of

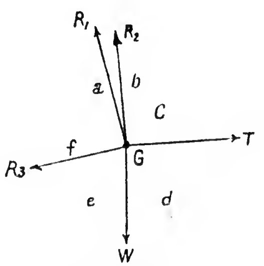


FIG. 105.

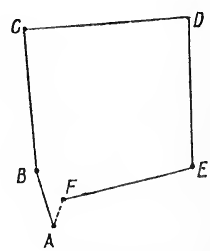


FIG. 106.

G from the lines of action of T, R_1, R_2, and R_3 respectively, and (*b*), the forces, represented in Fig. 105, acting at the centre of gravity.

If we now construct a diagram of forces (Fig. 106), we obtain AF, representing the resultant force acting through G.

The system of forces has, therefore, been reduced to a couple M, and a force represented in magnitude and direction by AF acting through G.

According to whether the value of M be positive or negative, so will the aeroplane rotate or tend to rotate in the clockwise or anti-clockwise direction respectively about an axis through G, while the action of the force AF is to cause the machine to rise or fall according to its direction, *i.e.*, according to whether its direction is as indicated in Fig. 106, or the opposite respectively.

For equilibrium and stability in the longitudinal direction, and preservation of the path of flight, M and AF must each be zero.

If the direction of R_3 and T pass through G, or if their moments relative to G cancel, we have the resultant couple :—

$$M = R_1 \times r_1 - R_2 \times r_2,$$

and if we take $R_1 = C_1S_1V^2\varphi$ and $R_2 = C_2S_2V^2\theta$ (see p. 10), where θ and φ denote the angles of incidence of the main and tail planes respectively, then for longitudinal stability we have :—

$$M = O = C_1S_1V^2\varphi \times r_1 - C_2S_2V^2\theta \times r_2,$$
$$\text{or } C_1S_1\varphi r_1 = C_2S_2\theta r_2,$$

i.e., angular equilibrium or longitudinal stability will be preserved, *independent* of the velocity of flight, so long as the relation represented by this equation holds. This relation may be realised by suitably inclining the movable plane to the direction of motion, and indicates the value of an auxiliary or tail plane in preserving equilibrium. The tail plane may consist of a fixed and movable part ; the latter is designated the elevator. It is obvious that each position of the elevator will correspond to a definite angle of incidence for any particular direction of flight, and consequently the longitudinal equilibrium will generally be disturbed with a movement of the elevator and the line of flight altered. It is not necessary that the elevator should be situated at the rear of the machine, but the effect of placing it aft is to bring about a slower or less abrupt movement of the aeroplane, a fact worth considering in view of the difficulty of maintaining a specific line of flight ; indeed, it may be said generally that only a *mean* straight path of flight is obtainable, the actual path pursued partaking more or less of a wave form, as indicated in Fig. 107, the amplitude of the wave being comparatively small.

Other conditions remaining the same, it will be seen that R_1 will increase in value as the direction of T is changed from a position above to one below the centre of gravity G, and *vice versâ*, and therefore the angle of incidence of the elevator will require to undergo a like change. The normal angle of incidence is the smallest angle that can be reasonably adopted without fear of danger to the machine, and since the latter

will be designed to be driven at some maximum speed, it follows that the normal angle of incidence corresponds with the maximum speed at which the machine may be driven, and will be less for a position of the line of thrust of the propeller above than for one below the centre of gravity. Obviously it will be advantageous to the machine and the aviator to have the position of the line of action of T above the centre of gravity, since the ultimate angle at which the elevator will require to be inclined to the fixed position of the tail plane when the motor is stopped during flight will be less than it otherwise would be.

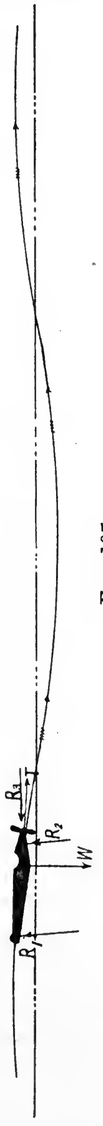


Fig. 107.

In what has gone before we have considered the effect of the various forces acting on the aeroplane in their relation to the axis of rotation passing through the centre of gravity, assumed to be a fixed point. In aeroplanes designed for military purposes, carrying perhaps munitions of war, which from time to time are discharged from the machine, the centre of gravity, and therefore the axis about which rotation is assumed to take place, will undergo change of position; and even in ordinary machines, where movement of the aviator or aviators on board takes place during flight, the centre of gravity will undergo change of position and in consequence will affect the equilibrium.

It will therefore be interesting to note the effect of a change in the position of the centre of gravity on the longitudinal stability.

Let Fig 108 represent the same forces acting on the machine as are shown in Fig. 104, except that R represents the resultant of the air pressure on the machine.

R may be such that, according to its position and direction, it will constitute either a positive or negative mechanical moment, tending to right or upset the machine, as the case may be, and represented by R . Gn.

Suppose G to be moved in the vertical direction along the line ab, through a distance denoted by d. The mechanical

moment will be represented algebraically by

$$R(Gn \pm d \sin \theta),$$

according as G is raised or lowered, where θ is the inclination to the horizontal of the perpendicular drawn from G to the direction of R.

Again, suppose G to move horizontally along the line *cd*, the

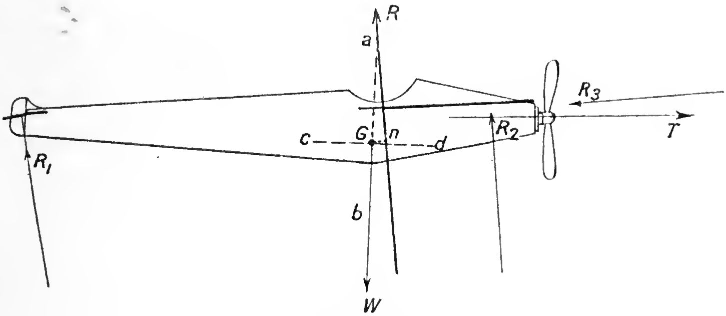


FIG. 108.

mechanical moment will be represented by the expression $R(Gn \mp d \cos \theta)$, according as G moves to the right or left.

The direction of R is usually as indicated in Fig. 108, in which case it will be seen that the effect of :—

(1) Raising G—is to increase the upsetting moment and decrease the righting moment.

(2) Lowering G—is to increase the righting moment and decrease the upsetting moment.

(3) Moving G forward—is to increase the upsetting moment and decrease the righting moment.

(4) Moving G aft—is to increase the righting moment and decrease the upsetting moment.

There is no real analogy between this case and that of a vessel floating in a buoyant fluid, although there is a similarity in the effect produced on the longitudinal stability. In the present case a vertical movement of the centre of gravity, if small, would produce little effect on the longitudinal stability, as in the case of a vessel floating in a fluid ; and a horizontal movement would tend to produce angular displacement in the clockwise or anti-clockwise direction, according to whether the centre of gravity moved forward or aft respectively, and in this

respect would correspond to a trim or change of trim by the head or stern respectively in the case of an ordinary vessel.

The lever or arm of the moment of the air pressure on the plane relative to the centre of gravity of the aeroplane under-goes little change with the usual *small* variation of the angle of incidence. This will be seen by considering Fig. 109.

Suppose the plane aB alters its position to $a\mathrm{B}_1$, so that the angle $\mathrm{B}a\mathrm{B}_1 = \psi$ is small. (Fig. 109).

Let R_1, R_2 denote the air pressures on aB, $a\mathrm{B}_1$ respectively. From what was stated on p. 10, it will be seen that the centre of pressure will change in position as ψ varies, moving towards the centre of area of aB, with an increase in the angle of incidence, as indicated in the figure; under such circumstances, since ψ is small, the perpendicular distances d_1 and d_{11} of G from the lines of action of R_1 and R_2 will practically undergo no change, and the moments of the air pressure will vary as the pressure, *i.e.*, as $c \cdot s \cdot v^2 \theta$, the normal pressure on the plane, where

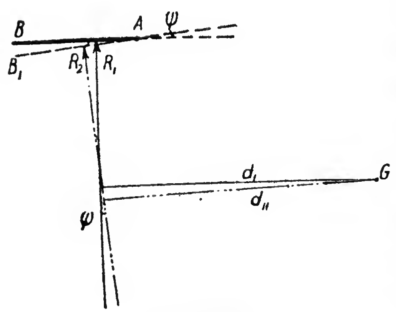


FIG. 109.

- θ is the angle of incidence (circular measure).
- c is a coefficient—dependent upon θ—the mean value of which may be taken $= \cdot 30$ for angles up to 7 or 8 degrees—see p. 10.
- s is the area of the surface in square metres.
- v is the velocity with which the air meets the plane in metres per second; and the pressure is measured in kilogrammes.

Applying this to the action of the elevator, the mechanical moment, and therefore the longitudinal stability, will vary as $v^2 \cdot \theta$, *i.e.*, inversely as the angle of incidence and directly as the speed, since the angle of incidence required diminishes as the velocity increases. In addition to this the effectiveness of the movable plane is enhanced on account of the considerable distance at which the tail is situated from the centre of gravity of the machine.

When an aeroplane departs from its normal position of equilibrium due to some disturbing influence, it should possess the power to return to that position quickly. In endeavouring to do so it will naturally be subject to some degree of longitudinal oscillation, which, however, may be checked or eventually damped out by the aid of the tail plane, situated as it is at the maximum distance from the axis of rotation at the centre of gravity, and consequently acquiring a considerable linear velocity as the machine oscillates.

The power of an aeroplane to return quickly to its normal position when disturbed has been designated *automatic* longitudinal stability.

Transverse or Lateral Stability.—The longitudinal elevation, or more correctly the projection at right angles, of the superficies of the aeroplane upon a longitudinal vertical plane, will possess a definite area, with relation to which we can obtain the locus of the centre of pressure and the pressure itself, as the angle of incidence varies from zero to 90°. This plane is used in calculations relating to lateral stability, and has been designated the vertical keel plane; it is situated in the plane of symmetry, is assumed to have a real existence, and to rotate with the aeroplane about a longitudinal axis passing through the centre of gravity.

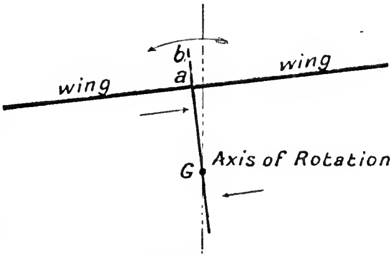


FIG. 110.

This axis, which has a definite direction and position in the machine, will vary in space, just as the centre of gravity, though fixed relative to the machine, will during flight mark out a curvilinear path in space, but the direction of the axis may not except coincidentally, be tangential to the path of the centre of gravity.

When the aeroplane inclines transversely—due possibly to the disturbing influences of air currents—a resistance is experienced on account of the resulting air pressure, the resultant statical moment of which relative to the axis of rotation will depend upon the area of the vertical keel plane

surface and the angular velocity of rotation. For stable equilibrium this moment must be a righting moment.

Obviously lateral stability may be increased by extending the vertical keel plane surface above the axis of rotation, *e.g.*, by extending it above the main plane or wings, as shown by *ab* in Fig. 110.

This has been tried, but has been discarded, for the obvious reason that the danger resulting from the effect of wind gusts acting on a surface situated at a considerable distance from

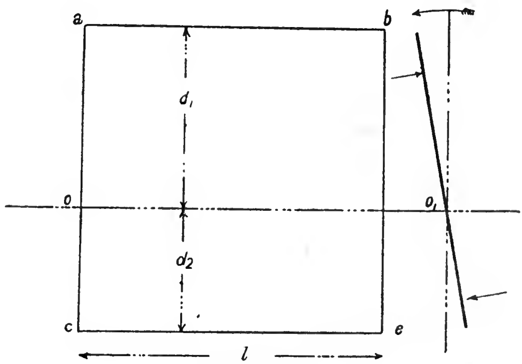


Fig. 111.

the axis of rotation more than counteracts the advantage of increased resistance to rotation.

The principle of the vertical keel plane surface may be illustrated as follows (Fig. 111) :—

Consider a rectangular surface *abce* rotating or tending to rotate about an axis oo_1 parallel to the edges *ab* and *ce* with an angular velocity $\frac{dw}{dt}$.

Let y be the distance of a strip taken parallel to the axis, of length l and width dy.

The normal resistance to motion of such a strip may be taken as :—

$$c \,.\, l \,.\, dy \,.\, v^2 \times \frac{y \cdot \frac{dw}{dt}}{v} = c.l.v. \frac{dw}{dt} \,.\, y \,.\, dy.$$

And its moment about the axis $= c.l.v. \frac{dw}{dt} \,.\, y^2 \,.\, dy,$

which integrated over the whole area of the plane gives :—

$$c.l.v.\,\frac{dw}{dt}\int_{d_2}^{d_1} y^2\,.\,dy = c.l.v\,\frac{d_1+d_2}{3}\,(d_1^2 - d_1d_2 + d_2^2)\,\frac{dw}{dt}$$

$$= \frac{c.s.v.}{3}\,(d_1^2 - d_1d_2 + d_2^2)\,\frac{dw}{dt},$$

where s denotes the area of the surface—an expression showing that the resistance to lateral angular movement depends upon the *first* power of the velocity of the wind and the *second* power of the linear dimension of the plane at right angles to the axis of rotation.

A vertical stabilising plane containing the axis of rotation will therefore constitute a couple of the form $m\,.\,v\,.\,\frac{dw}{dt}$, where m depends upon the form of the perimeter of the plane surface.

It is clear from this expression that the effectiveness of the plane will depend upon its depth ; but this very dimension must be limited, obviously on account of the danger which might result from inclination of the machine due to wind gusts.

Various devices have been resorted to, to bring into being increased resistance to rotation by causing a plane or planes normally horizontal to assume an inclined position.

Warping of the wings to obtain side balance is an example of this principle, the warping coming into action as the aeroplane moves laterally, and is controlled by the pilot. In this way not only is the air resistance to rotation of the machine increased, but the auxiliary force acting at a considerable distance—at the wing tips—from the longitudinal axis of rotation produces a considerable righting moment.

Farman adopted a flap hinged to the trailing edge of the wing (one at each end) which could be pulled down into the draught on the depressed wing tip ; a development of this method consists in making a portion of the wing itself to hinge.

" Warping," by which the entire wing is acted upon, consists in pulling the trailing corner of the depressed wing down into the wind, and is found effective in strong gusts and irregular winds. One result of warping the wing is to cause, or tend to cause, the machine to move in a direction different from that

intended; this has to be rectified by a suitable movement of the rudder—in fact, the action of the rudder and warping are interdependent.

There is one instance in which the machine is deliberately inclined transversely, viz., in taking a curve, the machine being sloped or banked inwards to prevent the tendency to slip down sideways or to outward " side slip " from the centre of curvature of the path of flight due to centrifugal action.

The size and shape of the wings differ according to the particular idea of the designer. The breadth, however, must of necessity be comparatively narrow relative to the spread, in order that the wings may as far as possible encounter virgin

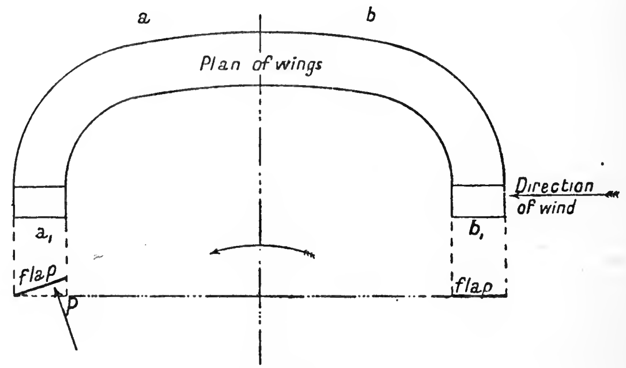


FIG. 112.

air, instead of air which, due to it having already met the forward or leading portion of the wing, has a movement downward. But there is a practical limit which the " aspect ratio," *i.e.*, the ratio of spread to breadth of wing, may assume, and is fixed, at least in part, by the consideration of efficient staying, since an increase in length of wing is accompanied by an augmented moment of air pressure on the wing about the middle line, or the longitudinal axis of rotation of the machine. In practice the ratio may be taken as about 6 : 1.

By superimposing one plane above another a grip of the air is obtained without unduly increasing the breadth of the wing; such an example is found in the biplane: little or no advantage, however, is found to result from using more than two planes.

Wildeblood's invention aims at maintaining balance in the machine by introducing flaps hinged at the outer ends of the wings; the latter are curved in such a manner that the flaps face inwards and are hinged about the outer edge of the wings, such that under the action of wind blowing across the machine, as indicated in Fig. 112, the tendency is for the wind to lift the wing b and so tilt the machine in anti-clockwise direction, at the same time passing over the flap b_1 but raising a_1; in this way a wind force P on the latter is produced, the moment of which counteracts or tends to counteract the tilting effect of the wind on the wing b. The action would of course be reversed for wind blowing in the opposite direction.

Laycock introduced the idea of a mass situated low and rigidly connected to the aeroplane by a sliding tube device as a means of obtaining horizontality in the longitudinal direction, and a modification of this was suggested by Major Renauld, by which the weight, in the form of a pendulum, was subject to certain forces, the one proportional to the head resistance of the aeroplane and the other to the propeller thrust, by mechanical means in such a way as to produce horizontality in the machine.

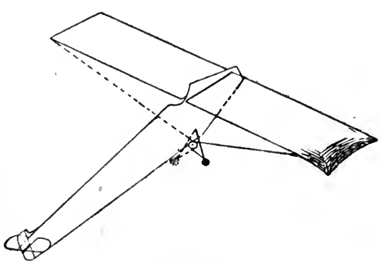


Fig. 113.

Another device is that of a pendulum or weight suspended from a point in the plane of symmetry, situated low and so arranged as to pull on the wing tip at that side towards which the weight has moved when the aeroplane inclines, creating or augmenting the air resistance on that side, tending to bring the wings again into the horizontal position and righting the machine.

Such a device (illustrated in Fig. 113) is, however, unworkable; the action would be too slow, and the power exerted by the pendulum too small, to be of practical utility.

Numerous other devices and inventions have been brought forward, most of which would be quite incapable of realising the results aimed at when practically applied, and it is very doubtful if any have really proved to be even reasonably

successful. There appears to be—at least for the present—a wide span between most devices which have been introduced, and which theoretically are good but practically are of little or no utility, and the actual realisation of some mechanical contrivance which, acting automatically, will relieve the pilot of the necessity of personal attention and so enable him to concentrate his energy and skill on the navigation of his machine.

The Report of the Departmental Committee appointed at the fall of 1912 to inquire into the causes of accidents to monoplanes of the Royal Flying Corps "urged the necessity of a general investigation into the stability of aeroplanes." They considered "the experimental data available was not sufficient to allow of a complete theory being formulated, that further investigation was needed and should be facilitated, and that the question of inherent stability should be examined, suggestions towards the solution of which have been given by the experiments of Lanchester and the calculations of Bryan."

They concluded that "the preference of monoplanes over biplanes was to be accounted for by the fact that the higher speed machine or monoplane was less disturbed by gusts and responded more easily to the wish of the flier."

Just as we have recourse to an elevator or horizontal rudder to act as an auxiliary to the main plane in changing the direction of flight in the vertical direction, so, similarly, is change of direction in the horizontal plane obtained by the use of a vertical rudder acting in conjunction with the vertical keel plane. In order to be effective it is necessary that the vertical rudder, which is of relatively small dimensions, be placed at a considerable distance from the axis through the centre of gravity about which the machine is assumed to turn.

In addition to the forces due to air pressure and gravity, there is created in turning an additional force due to centrifugal action which must be considered in relation to the stability of the aeroplane, in much the same way as was explained in the case of the submarine in Chapter IV.

Waterplanes or *Hydro-Aeroplanes* is the designation given to that type of machine which, when not in flight, rests in equilibrium on the surface of the water, but which when in flight may or may not be in contact with the water.

The history of the waterplane is of much more modern date than that of the aeroplane, and it seems strange that the former, entailing as it does less dangers, although perhaps presenting more difficulties than the latter, should have been so long in finding a solution, defective as that solution may be at the present time.

The waterplane may be described as a floating cylinder or cylinders, or floats surmounted by an aeroplane to which it is rigidly attached. The number of floats vary from one to more, according to the ideas of the particular designer.

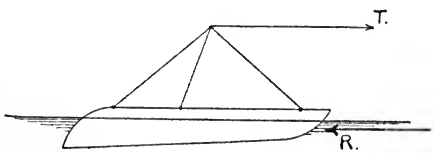


FIG. 114.

The chief points to be aimed at in shaping the design are:—

(1) Floatability.

(2) Minimum resistance when flying on the water surface or in the air.

(3) Stability when at rest or flying on the water surface or in the air.

(4) Good manœuvring qualities.

The question of floatability is not a difficult one to deal with, and means that, knowing the weight of that part of the machine surmounting the float, the latter requires sufficient buoyancy to support the whole machine. The question of resistance and stability, however, involves the consideration of shape or form to be given to the float.

The objection to boat-shaped floats is that at high speeds they offer greater resistance than do floats of the form of hydroplanes, which latter may be said to practically skim the surface of the water. Hence it is usual to shape the float more after the form of a hydroplane, at least at the after portion, the fore part being sometimes given a boat shape, but the bottom is practically flat all fore and aft. The reason for this is that as the machine is propelled ahead the tractive force T of the propeller forms, with the resistance R set up, a mechanical moment tending to bury the nose of the float in the water, and as the speed of the machine increases this tendency is also increased. The bottom of the float is therefore

made with a sloping plane bottom, as indicated in Fig. 114, so that with an increase of speed the pressure of water on the bottom of the float raises the fore end and so counteracts the effect of the propeller in tending to force the nose under.

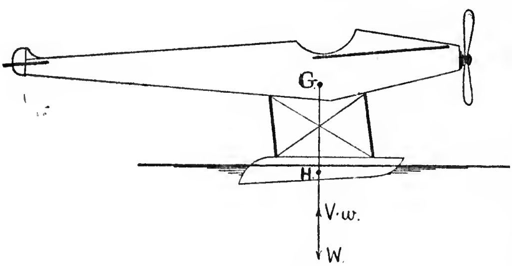


FIG. 115.

Expert opinions differ as to the minimum number of floats required, and in all probability it will eventually be found that three floats form the best arrangement, although some of the one-float machines have proved successful, not only in travelling on the surface, but in flying from and alighting on the surface of the water; whereas the last-named performance of alighting has not proved so easy or graceful an accomplishment as might be desired with machines possessed of more than one float.

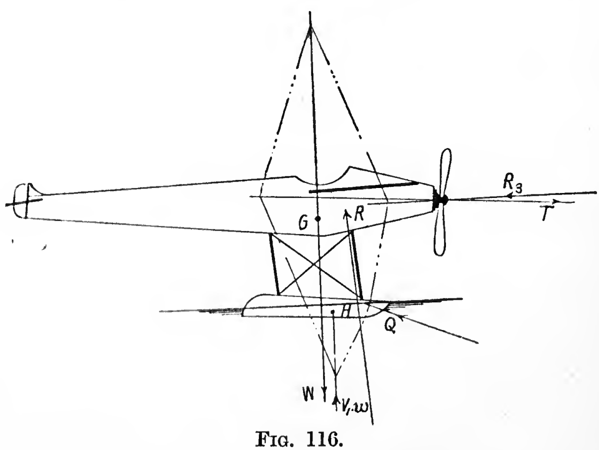


FIG. 116.

Whether the machine be of the single or multiple float type, the conditions for equilibrium when at rest on the surface of the water are that the centre of buoyancy H of the float (or floats) and the centre of gravity G of the machine are in the same vertical line (Fig. 115), and that the weight W of the

machine is equal to the weight $V \cdot w$ of water displaced by the float.

If the machine be started forward, as the speed is increased the centre of buoyancy H moves forward, due to the nose of the float at first being depressed, and a system of forces of

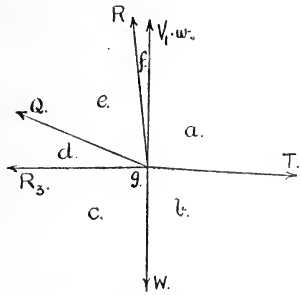


FIG. 117.

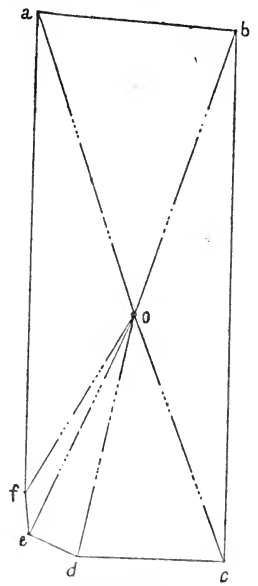


FIG. 118.

the nature of that shown in Fig. 116 is called into play, wherein the buoyancy $V_1 \cdot w$ is now less than the weight W.

R denotes the resultant air pressure on the planes.

R_3 denotes the resistance (other than R) of the air to the motion of the machine.

Q denotes the resistance of the water to the motion of the float.

T denotes the tractive effort of the propeller.

If we draw the direction of the forces from a common point g (Fig. 117), and construct a diagram of forces *abcdef* as shown in Fig. 118, then, if the forces balance, the figure *abcdef* will be a closed polygon and the funicular polygon shown by the broken lines in Fig. 116 will also be closed.

As the speed of the machine is increased the magnitude of Q will increase, and at high speeds will be such that the nose

of the float will be forced out of the water; the centre of buoyancy will move aft, and will presently occupy a position in which the buoyancy $V_2 \cdot w$ sets up a clockwise moment with respect to G; the disposition of the forces is indicated in Fig. 119.

As the speed approaches that at which the machine rises from the water, Q and $V \cdot w$ will undergo a great change, diminishing in value until, when the waterplane is on the point of taking to the air entirely, they each become zero, and the diagram of forces will be similar to that shown in Fig. 120.

With a single-float waterplane, unless the float be sufficiently

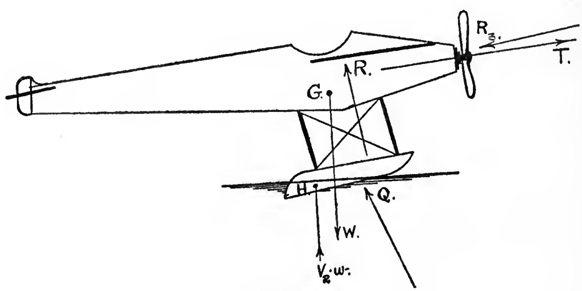


FIG. 119.

long and wide, neither the necessary longitudinal nor lateral stability can be obtained.

Instead of broadening the float, two narrow floats placed parallel to each other at a considerable distance apart is much more efficient from the point of view of lateral stability, and a third float of small dimensions placed under the tail will if necessary augment the longitudinal stability should the tail fall. In some cases a single main float with small auxiliary floats or balances under the wing tips, which only come into action upon the machine tilting laterally, are used.

There are difficulties to be found with all types, and the thing is to determine the best arrangement, not altogether from a theoretical standpoint, but from a judicious consideration of theory combined with actual experimental results.

With the two-float machine, unless both of the floats leave or touch the water at the same time, the machine tends to rotate horizontally, and is likely to bring severe strains to bear

upon the structure of the machine. For the same reason, in the case of the single float with auxiliary balancers at the wing tips, if one of the latter suddenly dips into the water due to the machine tilting while flying on the surface, the wing to which it is attached is likely to be broken or badly damaged.

The Dirigible, or Airship, consists of a body of gas enclosed in an elongated strong covered framework termed the "hull," supporting one or more cars for carrying the load, the motors for driving the propellers, and surmounted by planes for the purpose of navigating the ship in the longitudinal vertical direction and steering in the horizontal. Side fins may also be used to check the rolling of the ship should such motion be set up.

The mechanical principles applied in considering the equilibrium of the aeroplane are equally applicable here, but now we have to consider the buoyancy of the airship together with the considerable resistance to motion due to the immense surface of the hull envelope presented to the air. Both of these forces are calculable quantities, and hence we can estimate with some degree of accuracy the conditions under which the airship will be in equilibrium.

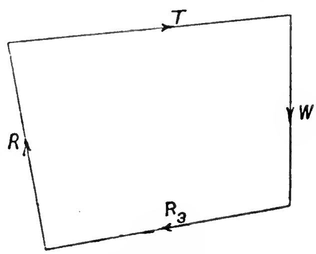


FIG. 120.

With reference to resistance, the expression $Csv^2\theta$ would be used in the calculation, carefully applying it to the whole of the exposed surface.

The cubic content of the parts forming the structure may be neglected in comparison with the volume of the gas-holding compartments. Hence, if we assume W to be the weight of the airship less the contained gas, acting vertically downwards through the centre of gravity, and V to be the volume enclosed by the envelope, the resultant force due to displacement of air acting upwards through the centre of volume of the envelope, *i.e.*, through the centre of buoyancy, will be represented by $V(\rho - \sigma)$, where ρ and σ are the densities of the air and gas respectively, say in lbs. per cubic foot.

Imagine the airship floating at rest in still air, the centre of gravity and centre of buoyancy would be in the same vertical

line, the latter point being situated above the centre of gravity, and $W = V(\rho - \sigma)$.

This is a condition which is not usually realised in practice. Suppose the airship to be travelling at a velocity v, and let

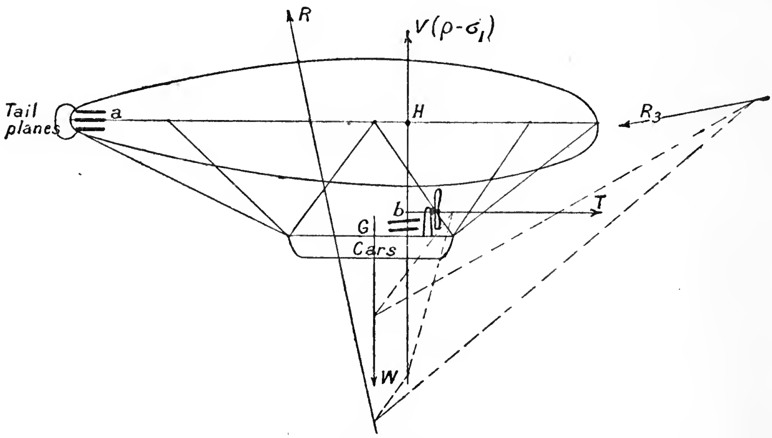


FIG. 121.

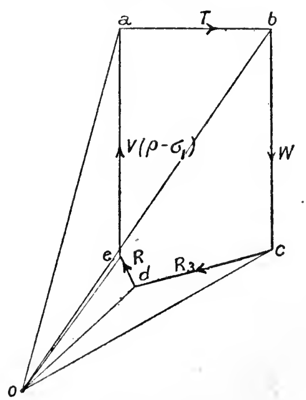


FIG. 122.

R denote the resultant air pressure on the planes at a and b (Fig. 121), R_3 the resistance (other than R) of the air, and T the tractive force.

The disposition of the forces will be somewhat as indicated in the figure. For equilibrium in the longitudinal direction the polygon of forces must close, as shown in Fig. 122, and the

funicular polygon, shown by the broken lines in Fig. 121, must also close.

Generally the centre of buoyancy H and the centre of gravity G are not in the same vertical line for the horizontal position of the airship, the centre of gravity being situated a little abaft the centre of buoyancy.

Having decided on the shape and size of the hull, we can, by imposing the conditions for stable equilibrium just cited, determine the position of the centre of gravity and distribute the weight of the car, equipment, and load to be carried accordingly.

The necessity of planes for stabilising airships is now recognised, and one of their functions is to damp out oscillations which may be set up through the equilibrium being disturbed, and so produce a steady platform in the car of the vessel. The action of the rudder used in steering will be identical in principle with that of the aeroplane, while the horizontal movable planes, such as are indicated in the figure at *a*, are used to bring about a change in altitude.

For lateral angular disturbance of the airship the power of the planes at *b* to check the disturbance will depend upon their area and length, and the velocity of roll. The chief factor, however, is that part of the inherent stability represented by the joint action of the weight W and the buoyancy $V(\rho - \sigma_1)$. Let D be the distance between the centre of buoyancy and centre of gravity, and θ the angle at which the airship at any time is inclined, the stability represented by the action of W and $V(\rho - \sigma_1)$ resolves itself into a force $W - V(\rho - \sigma_1)$ acting vertically at G and a righting couple $V(\rho - \sigma_1)D \sin \theta$.

When turning from the linear to, and moving in, a curvilinear path there is the additional influence of the centrifugal force on the stability ; this may be considered in a manner similar to that in the case of the steering of a submarine when wholly immersed, dealt with in Chapter IV.

CHAPTER VI

CAISSONS

Dock engineers are not uniformly agreed as to the precise relative merits of caissons and gates for closing dock entrances; whether in any specific instance the one may be more advantageous than the other does not concern us here, as the question of dock gates does not enter into the present discussion. There is, however, one particular case which might be mentioned in passing where the gate takes the form of a flap, working about a horizontal axis at the sill of the dock, being vertical when the dock is closed, and is turned through an angle a little greater than 90° when the entrance is open; it will not be necessary, however, to make any further reference thereto.

The chief features of the caisson are that by its means the upper deck may be utilised as a footway or roadway across the dock entrance; that it may be hauled up on to a slipway, or docked for repairs and painting with considerable facility, much in the same way as in the case of an ordinary vessel.

Caissons may be divided into two classes, viz.:—

(*a*) Floating or ship caissons.

(*b*) Sliding or rolling caissons (which also may float).

Those of the former class may be so designed as to be available for closing several entrances independently and singly, or to split up the length of a long dock to accommodate the particular vessel to be docked, and for which the full length of the dock would not be necessary.

Caissons of the second class invariably occupy one position at the dock entrance, and are usually only moved out for repairs or survey.

All or some of these points must necessarily be considered in working out the design, and will very largely govern the weight of the structure; this latter together with the outward form or shape given to the caisson are the principal factors determining the stability.

Another factor of paramount importance is that water-tight

compartments and water-tight platforms or decks have to be worked into the caisson, apart from considerations of strength, in order that the necessary provision for carrying the fluctuating or water ballast may be made. In this respect there is a difference between a ship and a caisson, inasmuch as in a ship decks and bulkheads are provided primarily for strength, and additionally, when water-tight, to aid her stability by enclosing water-tight spaces or compartments, which, if necessary, may be used for carrying water ballast, or in the event of the vessel being bilged will confine the water flowing in from the sea. With a caisson it becomes absolutely necessary to provide water-tight compartments, for the reason that water ballast is required to work the caisson, but the water-tight flats and bulkheads so worked into the structure are not necessarily required for strength, although they naturally do add strength to the structure.

The caisson generally may be described as a vessel of box shape, or of modified box form, so designed as to be capable of flotation under certain conditions of draught or water level, dependent almost entirely upon the depth of water at high or low water level and the range of the tide; it must be so designed, and if necessary additionally ballasted, as to be stable under these conditions.

The simplest case obviously consists in a water-tight hollow vessel which may be sunk by admitting water into and raised by pumping water out of it.

In practice, however, the caisson is divided into several compartments, one or more of which is used for carrying the permanent and fluctuating ballast, the remainder being preserved as air chambers to supply the requisite amount of buoyancy when the caisson is being floated from the stops.

Having decided upon the type of caisson, roughed out the section, assigned the scantlings and derived therefrom by calculation the weight and centre of gravity of the structure, the amount of permanent ballast would be estimated necessary to give the requisite amount of stability for this—light—condition of flotation.

Two other conditions must now be considered, viz., those relating to the low and the high water levels. The quantity of water ballast required to ensure the caisson resting safely

in position in the grooves without rising from its seat is estimated, and the compartments for carrying the ballast so arranged, both with regard to capacity and position, as to secure a suitable amount of stability should it be necessary to float the caisson at either state of the tide.

Taking a caisson of ship form, the vessel would generally be divided into three compartments by two water-tight decks or flats in addition to the top deck or platform. Such a caisson is illustrated diagrammatically in Figs. 123 and 124.

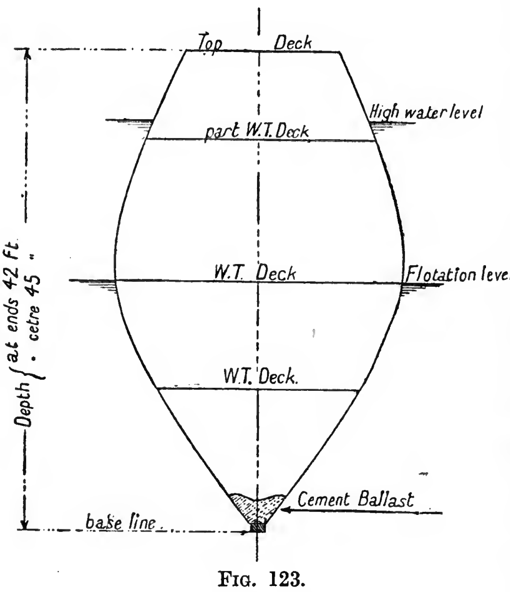


Fig. 123.

The lowest compartment contains the permanent ballast and a portion of the fluctuating water ballast, while the middle compartment serves as an air chamber, the buoyancy of which is the chief factor employed to raise or float the caisson; this compartment is often sub-divided, one of the subdivisions being utilised for water ballast.

The highest compartment is fitted with flood-holes, enabling water to find ingress or egress with the rise or fall of the tide, as the case may be, when the vessel is resting in position. The flood-holes may be closed by sluices when required to exclude the water, and are so closed on the inner or dock side of the caisson when the latter is in position in order to prevent the external water finding access to the dock.

It follows that, provided the caisson will not lift when the second or tidal deck is awash, there will be no tendency for it to rise from its seat with a rising tide.

Immediately under the upper deck a tank (or tanks) is

constructed for carrying water ballast, the absence or presence of which is intended to form the balance of weight necessary to cause the caisson to rise from or keep its seat at the lowest flotation level—or for small draughts.

To sink the caisson water is admitted to the lowest compartment and the top tank is filled, or partially filled with water if necessary, or for small draughts the filling of the top tank alone may be found to be sufficient.

By allowing the water to escape from the top tank, or by pumping water out of the lower compartment, the caisson may be caused to rise, the amount of lift required in order to

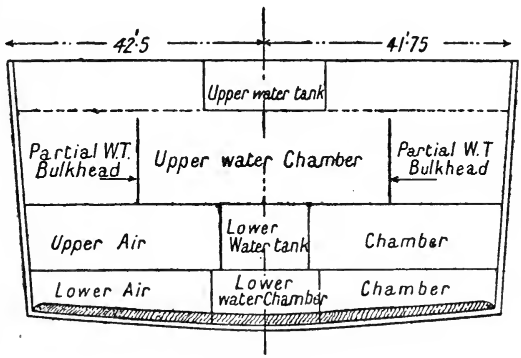


FIG. 124.

allow the caisson to clear the grooves being dependent upon the amount of batter of the sides at the entrance.

In the uppermost chamber two partial bulkheads are fitted to prevent any sudden flow or transference of water towards either end of the caisson should it tilt or rise unevenly from the sill.

If, when the caisson is floating freely with the tidal deck awash, or if the caisson be in position under the same condition, the volume of displacement is practically represented by the capacity of the vessel below the tidal deck; and the caisson may be compared to a totally submerged body; the centre of buoyancy and metacentre are coincident, and for stable equilibrium it is necessary that the centre of gravity shall fall below the centre of buoyancy.

In practice the distance between these points is usually about 12 to 18 in. for the tidal-deck-awash condition; but

generally the amount will depend upon, and must be an adequate amount for, the particular case considered.

In the case illustrated by Figs. 123 and 124, when the caisson is floating freely with no water ballast, *i.e.*, in the light condition, the centre of gravity is situated about 11 in. below the centre of buoyancy and the metacentric height is 4 ft. $7\frac{1}{2}$ in.

With 20 tons of water in the lower water chamber and 55 tons of water in the lower water tank the tidal deck becomes awash and the centre of gravity is situated about 17 in. below the centre of buoyancy, *i.e.*, the metacentric height is 17 in., since the centre of buoyancy and metacentre are coincident. With the floodholes closed and the same condition of ballasting maintained the metacentric height would be increased to nearly 5 ft.

The distribution of weight and the metacentric height under these conditions are shown in the tabular calculation set out below :—

(1) Light condition.

	Tons.	C.G. above base in ft.	Moments.
Weight of caisson and permanent ballast .	706	12·5	8825
Centre of buoyancy above base . .		13·4 ft.	
,, gravity below C.B. . . .		·9 ,,	
Metacentre above base		17·12 ,,	
Metacentric height (light condition) . .		4·62 ,,	

(2) Tidal deck level condition.

	Tons.	C.G. above base in ft.	Moments.
Light condition	706	12·5	8825·0
Water in lower water chamber . .	20	7·0	140·0
,, ,, ,, tank . . .	55	15·7	863·5
Displacement at tidal deck level . .	781	12·58	)9828·5
Flood-holes open. C.B. above base. . .		14·0 ft.	
Flood-holes open. C.G below C.B. (metacentric height). . .		1·42 ,,	
Flood-holes closed. Metacentric above base .		17·55 ,,	
Flood-holes closed. Metacentric height . .		4·97 ,,	

With the flood-holes closed the caisson would float at the high-water level of 39 ft. 0 in. above base, with a displacement of 1,770 tons; this displacement is made up as follows:—

	Tons.	C.G. above base in ft.	Moments.
Light condition	706	12·5	8825·0
Water in lower water chamber .	20	7·0	140·0
,, ,, ,, tank .	55	15·7	863·5
,, upper ,, chamber* .	944	28·5	26904·0
,, ,, ,, tank .	45	41·0	1845·0
Displacement at high-water level.	1770	21·79	)38577·5
Centre of buoyancy above base		22·70 ft.	
,, gravity below C.B. .		·91 ,,	
Metacentre above base . .		23·66 ,,	
Metacentric height . .		1·87 ,,	

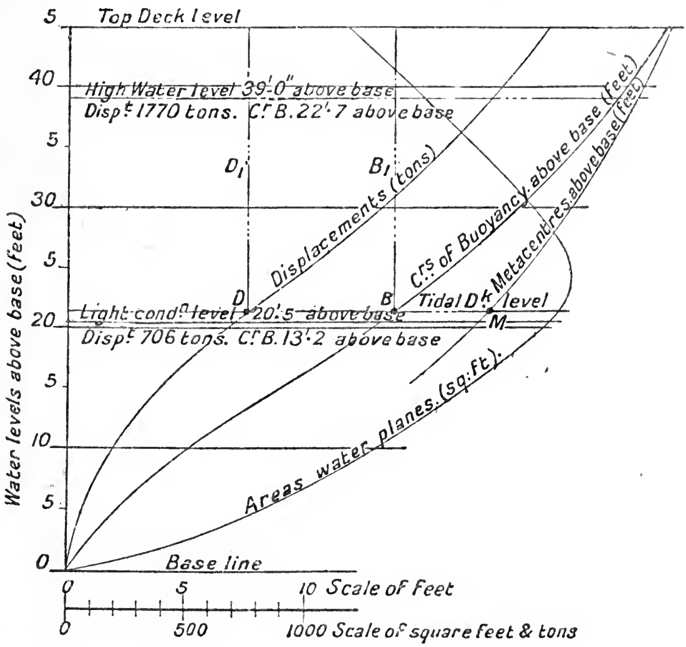


FIG. 125.

The 39 ft. water level is the height to which the water would reach at high tide when the caisson is resting in position at

* Height of water in this chamber 12.8 ft.

the dock entrance. It is obvious, therefore, that in order to ensure the caisson resting properly in the grooves water ballast a little in excess of that given above would be required in order to prevent any tendency to rise on account of a possible fluctuation of water level due to a rough water surface.

To remove the caisson from the dock entrance sufficient water is allowed to escape from the top tank or is pumped out of or ejected from the upper water chamber to cause the caisson to rise clear of the grooves ; the amount of rise or lift required in this case is 4 ft., and it is clear that the caisson should possess a suitable amount of metacentric height when floating at the water level of 35 ft. above base.

The case for this condition is worked out below :—

	Tons.	C.G. above base in ft.	Moments.
Deductions :—			
Water in upper water tank .	45	41·0	1845·0
" " " chamber	140*	31·8	4452·0
	185		6297
Displacement at 39 ft. water level	1770		38577·5
" " 35 ft. " "	1585	20·36	)32280·5
Centre of buoyancy above base		20·95 ft.	
Centre of gravity below C.B. .		·59 "	
Metacentre above base . .		22·42 "	
Metacentric height . .		2·06 "	

Fig. 125 shows diagrammatically the variation of displacement, water plane area, and height of centre of buoyancy and metacentre above base, with the height of water level above base.

The diagram needs no further explanation except that it may be observed that at the tidal deck level and with the flood-holes open the curve of metacentres breaks away horizontally from M to B—the centre of buoyancy for this condition; and for water levels greater than this the metacentre and centre of buoyancy remain coincident and situated at a constant height above base represented by the vertical line BB_1. The displacement also would be constant for water levels greater

* 140 tons taken from the central compartment between bulkheads.

than the tidal deck level, and is indicated in the diagram by the vertical line DD_1.

The type of caisson considered above is useful where there is no big range of tide level or where the caisson is used to close the entrance to a dock constructed to accommodate vessels of fairly deep draught.

It is clear from the diagram shown in Fig. 126 that in order to float the caisson out or in, the depth of water over the sill of the dock must be at least 23 ft. 9 in., in which case the draught of the vessel should not exceed 23 ft. 3 in., allowing 6 in. for clearance; and the maximum draught of vessels may therefore vary between 23 ft. 3 in. and 37 ft. 9 in. (= 39 ft. 0 in., less 9 in. groove, less 6 in. clearance), according to the state of the tide, where 39 ft. 0 in. is the depth of water above base at high-water level.

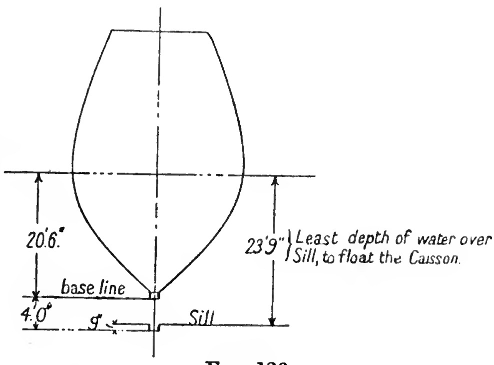


FIG. 126.

Such a caisson would be designed for docks specially built to accommodate large passenger vessels and warships of deep draught, but other vessels of comparatively light draught would naturally not be precluded from using such a dock, the only objection being the greater amount of time required to let the water into and pump the water out of the dock than would be the case with one of more suitable dimensions.

If the conditions were such that at low-water level the depth of water over the dock sill were small, and that it is necessary to provide for docking vessels of *small* draught with expediency, the caisson would then be required to be capable of flotation at a small draught as well as at any state of the tide between the high and low water levels.

To meet these conditions the shape of the caisson would be so modified as to possess relatively a greater amount of buoyancy at the lower water levels than in the case just

considered, the internal arrangement of water and air chambers being much the same as in the case just cited.

An example is found at the Bute Docks, Cardiff, and is illustrated in Fig. 127, showing the transverse section of the caisson.

The shape is peculiar, the horizontal section of the lower portion being much greater than that at the upper part of the caisson. The bell shape was adopted in order to obtain the necessary amount of buoyancy and initial stability for the caisson to be floated at small draughts with safety.

To move such a caisson when floating freely and possessing a large freeboard involves an element of risk which it is well to reduce to a minimum; and the danger of movement is always enhanced where the caisson occupies an exposed position and perhaps is subject to a strong wind force. For this reason the question of stability has to be carefully considered.

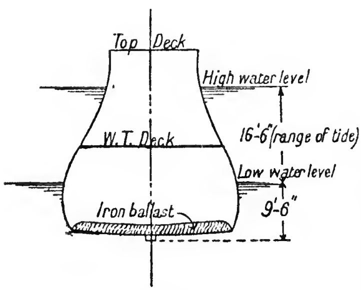


Fig. 127.

A caisson similar in construction to the Bute Docks caisson is that designed by Sir John Wolfe Barry for the Barry Docks. In this case one of the sides is ship-shape while the other is flat and vertical, this latter forming the inside surface, while the former is always exposed to the fluid pressure.

The reason of the flat surface is that the caisson was designed to suit several positions so as to be capable of fitting against the masonry at any one of the several entrances to the dock or basin and when not in use to be housed in a recess provided for it with as great economy of space and as much facility as possible.

This caisson floats with a draught of 18 ft. 6 in., and with some 350 tons of permanent iron ballast weighs about 700 tons. Sections of the caisson are shown in Fig. 128.

When the caisson is in position the valve to the upper water chamber stands normally open, allowing the water in the chamber to take the level of the water outside. If the caisson is to be moved the water in the top tank is run off and the

valve to the water chambers closed, when these latter are emptied by hydraulic ejectors, the caisson floating at a draught

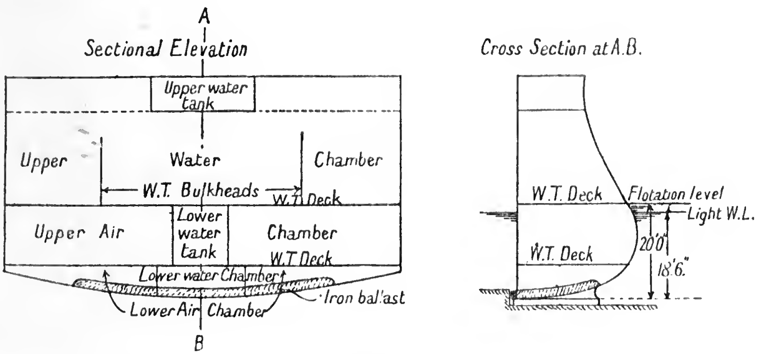


Fig. 128.

or level of 20 ft. The caisson is fitted with various ingenious appliances for keeping it in trim when in this condition.

Caissons of somewhat unusual design were constructed for

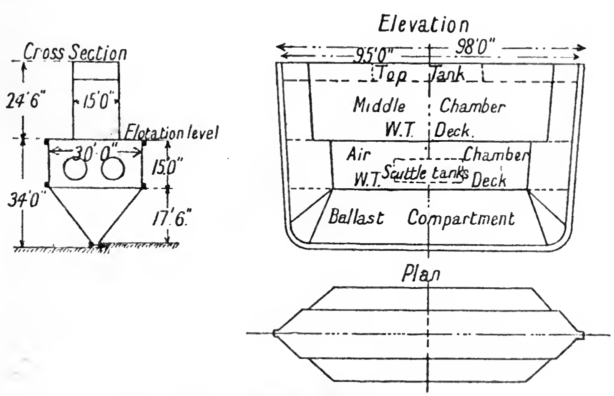


Fig. 129.

H.M. Keyham Dockyard extension; their form may be described as between the simple box and the ship-shape caisson.

Fig. 129 shows diagrammatically the shape of the caisson. It will be noticed that midway between the ends of the caisson the vertical section through the upper water chamber and the air chamber consists of two rectangles, and below the first water-tight deck from the bottom the section is triangular.

The rectangular sections of the upper water chamber and the air chamber are preserved uniformly to within about 10 ft. and 15 ft. respectively of the stems; thence to the ends these chambers are triangular in plan.

In considering the stability of this caisson it is necessary to understand the method adopted in regulating the water ballast during sinking or raising operations.

The caisson is divided into five main chambers, viz. :—

The bottom ballast compartment.

The air chamber containing the scuttle tanks.

The middle upper water chamber containing the top tank.

The two end chambers above the second water-tight deck.

The scuttle tanks are filled from outside through 9 in. valves in the sides of the tanks, and are emptied by blowing out into the top tank by means of compressed air or by hand pumps. The top tank is emptied by gravitation into the middle chamber, which latter is in communication with the external water. The

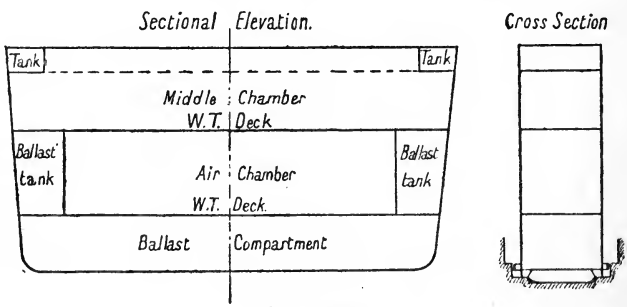


FIG. 130.

filling and emptying of the end chambers is regulated by 18 in. valves, and by this means the trimming of the caisson is controlled.

Suppose the caisson in position with all tanks full and required to be raised. The valves in the end chambers are opened and the scuttle tanks are blown out into the top tank; thence the water is allowed to gravitate into the middle chamber, and the caisson rises gradually from the stops until it floats at the level of the top deck of the air chamber.

The caisson selected for illustratiom weighs approximately 1,000 tons when fully ballasted and with all water tanks empty, *i.e.*, in the light condition.

Sliding and rolling caissons are not so varied in design as the floating caisson, and perhaps are more easily worked and housed, especially under adverse weather conditions. Generally they are of box or rectangular form, and may slide, roll, or—if buoyed off their seat—float into or from the recess specially constructed to receive them at the dock entrance, according as the latter is required to be opened or closed.

They are divided into two or more compartments or chambers and sub-divided much in the same way as floating caissons.

Fig. 130 represents a sliding caisson divided into three main compartments by two water-tight decks, the middle compartment being an air chamber, in which at the ends are provided two water ballast compartments used as emergency tanks at high-water levels. Under the top deck of the caisson is also provided two water ballast tanks, one at each end of the top compartment, and are used under extreme tidal conditions to ensure an excess of weight over buoyancy.

There is another function which these tanks have to perform. The caisson is hauled into or out of the recess on keels sliding over granite paths by means of chains attached at points situated about 3 ft. 6 in. below the top deck level. The force of 35 tons exerted on the chain will move the caisson across the entrance in about four minutes, and is such that there is a tendency to tilt the caisson longitudinally about the lower edge of the forward end during the travel ; this tendency is provided against, or at least lessened, by filling the far upper water ballast tank, thus providing a force-moment about the above-mentioned edge, keeping the caisson in its normal or upright longitudinal position.

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